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ANALYSIS OF UNIT BREAKPOINTS IN LAND COMBAT

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Naval Postgraduate School
Monterey, California

March 1975

NAVAL POSTGRADUATE SCHOOL

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THESIS

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by

Richard C. Adkins

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Thesis Advisor: J. G. Taylor

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Analysis of Unit Breakpoints in Land Combat

by

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Captain, United States Army
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ABSTRACT

The thesis considers battle termination (a unit reaching its so-called "breakpoint") in ground combat as a rational decision process. A commander's decision to break contact with an enemy force and withdraw from the battlefield is analyzed for company-size infantry units. Two approaches for modelling a commander's decision process to terminate an engagement are presented. The first approach is based on extrapolation of observations on past battle history into the future with no assumption about combat dynamics. The second is based on the assumption of known Lanchaster-type combat dynamics (possibly with unknown parameters to be estimated) and uses Kalman filtering. Possible applications of such models are discussed, and related areas for future study are recommended.

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I. INTRODUCTION

In the past there have been many attempts to model land combat. Two reasons for attempting to model this process are: (1) to learn more about the actual dynamics of land combat and (2) to aid in predicting outcomes of possible future conflicts. The models that have been developed vary greatly in both type and complexity. Some of the more recent models, with the aid of digital computers, are capable of considering many variables in great detail [Refs. 9, 10, 17, 35, and 36]. As the trend toward more complex and detailed models progresses, even more subjective variables such as suppression and intervisibility are being quantified and included in models. With all the emphasis on more realistic and detailed models there is one area that has been somewhat neglected.

A neglected area in the modelling of land combat is the establishment of criteria for battle termination. In many models the only criterion for terminating a battle is the fractional-casualties suffered by the participants [Ref. 17]. Other models are even less sophisticated in that the criterion for battle termination is the annihilation of one of the opposing forces. The reasons for using casualties as the sole criterion for battle termination are not completely known but some of the reasons will be discussed later.

In actual battles several different events might cause battle termination. If one of the opposing forces is

annihilated, then the battle ends; however, this is a rare event essentially never observed in combat with a few exceptions (e.g. Iwo Jima, Alamo, etc.). If one of the opposing forces surrenders unconditionally, the battle will also terminate. The third and most common event which will result in battle termination is that of one opposing force breaking contact with the enemy and withdrawing from the battlefield [Ref. 41]. Naturally there are other possible events which might result in battle termination, but they will not be considered here.

The event of particular interest in this analysis is that of one opposing force breaking contact with the enemy and withdrawing from the battlefield. There are numerous reasons [Ref. 30] why this event might occur and several are as follows:

1. To draw the enemy into an unfavorable situation.
2. To permit the use of the unit elsewhere.
3. To gain time.
4. To conform to movements of friendly troops.
5. To shorten lines of communication.
6. To avoid further combat.

Of the six reasons listed the only one this analysis will consider is the last one.

At this point it is worthwhile to introduce the concept of a breakpoint. In military literature there are several different definitions for the term breakpoint. For the purposes of further discussion a breakpoint is defined to

be that state of a battle which a unit considers itself no longer capable of performing its mission and as a result elects to break contact with the enemy and withdraw from the battlefield. Therefore, when a unit withdraws from the battle strictly to avoid further combat, the unit is considered to have reached its breakpoint. A unit might never reach its breakpoint in some tactical situations that do not allow for withdrawal. There might be other reasons (i.e., physical constraints or lack of morale) why withdrawal is unlikely, and the unit might elect to surrender, but those situations are not within the scope of this thesis.

With the given definition of breakpoint in mind this analysis will investigate essentially the following three questions:

1. When does a unit reach its breakpoint?

2. What are some of the significant variables that cause a unit to reach its breakpoint?

3. What approaches might be taken to model a unit's breakpoint?

The motivation for gaining insight into the breakpoint is to provide new approaches to modelling the battle termination process. The reasons that the modelling of battle termination is important are: (1) battle termination sub-models are widely used in large unit combat models such as ATLAS and VECTOR-II [Refs. 17 and 35] and, (2) such sub-models may have a major influence on optimal time-squential tactical allocation strategies [Refs. 32 and 33].

II. MODELLING THE BREAKPOINT AS A FUNCTION OF FRACTIONAL CASUALTIES

In many current land combat models a unit's breakpoint is determined by the percentage of casualties sustained by that unit [Refs. 17 and 35]. Two inherent assumptions in these models are that a unit's effectiveness decreases as the number of casualties increase and a unit becomes more likely to reach its breakpoint as it's relative effectiveness becomes less. Both of the assumptions seem intuitively appealing and would certainly be agreed upon by the majority of the military community. The point of contention, however, is the explicit relationship between unit effectiveness, casualties, and the breakpoint. Also it might be proper at this point to question the existence of such a relationship without consideration of other intervening variables. The discussion in this section will center around the relationship between battle casualties and unit effectiveness and the validity of present breakpoint hypotheses.

A. BATTLE CASUALTIES AS A MEASURE OF LOSS OF UNIT EFFECTIVENESS

There have been numerous studies on battle casualties from past wars. In many cases the studies were done to gain insight into the casualty process itself. One such study was done by Beebe and DeBakey [Ref. 1] on battle casualties suffered by the U.S. Army in WW II. They listed the following variables as being significant in producing variation in casualty rates:

1. Ratio of enemy to U.S. strength

2. Weapons employed, and ratio of enemy to U.S. fire power

3. Experience and training of troops, both in general and in particular types of combat

4. Terrain

5. Tactical advantage and excellence of plan, enemy and U.S.

a. Availability of prepared positions

b. Possession of terrain advantages, e.g. high ground

c. Intelligence

6. Tactical and strategic support, both air and naval

7. Logistic support

The above list illustrates the fact that the number of casualties suffered by a unit during any period of time is dependent on numerous variables. This suggests that it is not sufficient to talk about only the number of casualty suffered by a unit without putting that number into the context of a time frame and tactical situation. Certainly, one would not equate equally the combat effectiveness of an infantry platoon which had suffered one casualty per day for a period of ten days from sniper fire to that of the same platoon which had suffered ten casualties in a period of five minutes from a charging enemy company. If one is forced to relate casualties and unit combat effectiveness only in the context of a specific time frame and tactical situation then it is legitimate to ask if there are any

general relationships which are universally applicable. In a study done by Best [Ref. 3] three general conclusions were cited:

> 1. Casualties are essential yet variously contingent determinants of combat, for they tend to diminish, constrain, depress, or derange the adaptive application of force to differing degrees in different situations; therefore, they (a) reduce the tempo of tactical development to varying extents, and (b) exert a varying influence on the tactical outcome - disproportionate and decisive, proportionate and substantial, or none at all.
>
> 2. Casualties are a qualitatively, but not quantitatively, predictable diffuse depressant in overall operational effect.
>
> 3. Quantitative regularities in aggregated casualty rates are mainly expressions of the prevailing intensity of combat. Although in part determined by casualties, prevailing intensity is in greater part determined by other constraints and restraints on the functioning of tactical systems: uncertainty and risk; delays and deficiencies in communications, and logistic insufficiencies.

Although the conclusions reached by Best provide no explicit relationship between casualties and unit effectiveness, they do emphasize the variance that exists in the effect of casualties on operational effectiveness. It would appear that any quantitative analysis on the relationship between casualties and unit effectiveness is, at least, partially restricted to specific types of situations. This would require knowing what variables are necessary to classify a type of situation and the value of these variables for each specific situation. For example, it might be important in performing the analysis to know (a) whether the unit in question was attacking or defending, (b) the amount of training received, (c) the terrain occupied and fired over, and (d) the means available to evacuate

casualties. These variables are only a few of the many variables which could be considered in trying to establish a "type" of situation.

The danger of specifying too many variables arises. If it requires thirty variables to categorize a "type" of situation then each situation would be a unique "type" and quantitative analysis would not be feasible. Another approach to accessing the loss of combat effectiveness due to casualties is a subjective one. This approach entails the collection of expert opinion from men experienced in land combat. The subjective opinions of military men on how effective a unit is with varying casualty percentages would have to be consolidated to form an estimate. This type of an approach was taken by Spring and Miller [Ref. 29] when they developed a graph depicting the "Relationship between an attacking infantry company's percentage casualties and the percentage of its surviving riflemen that are ineffective . . .". There are also some graphs in FM105-5 [Ref. 21] that show the relationship between percentage casualties and ineffective time for attaching and defending units. At present it appears that one is forced either to refer to a specific combat situation or to accept estimates based on military judgement when one wishes to quantify the loss of combat effectiveness due to casualties.

B. PRESENT METHODOLOGY FOR MODELLING BREAKPOINTS

With the realization that not many land battles are fought until one side or the other is annihilated, many

analytic combat models and computer combat simulations have been provided with rules for terminating the battle. The most common rule is to assign (either stochastically or deterministically) both sides a breakpoint based on a casualty fraction value [Refs. 17 and 35]. This implies that when a unit suffers a fraction of casualties equal to its breakpoint state the unit becomes ineffective (either surrenders or withdraws from the battlefield) and the battle is stopped. Usually the side which reaches its breakpoint first is considered to be the loser. The casualty fraction is defined as $(1 - \frac{x(t)}{x_o})$ where x_o is the initial number of combatants in unit X at the start of the battle and x(t) is the number of remaining combatants (i.e., non-casualties at some time t during the battle). If the breakpoint is defined as B (where $0 \leq B \leq 1$) then the rule for unit X may be simply written as: If $(1 - \frac{x(t)}{x_o}) \begin{array}{l} <B \text{ continue fighting} \\ \geq B \text{ stop fighting} \end{array}$.

An assigned breakpoint B might never be reached exactly because casualties occur in a discrete manner, and this is the reasoning for the $\geq$ sign in the stopping rule. Associated with this type of model are break curves. Figures 1 and 2 are examples of two types of break curves [Ref. 13]. Figure 1 is a deterministic break curve and consists of a step function at a casualty fraction value of .2. This implies that with certainty of probability equal to 1.0 that the unit will stop fighting (or reach its breakpoint) when $(1 - \frac{x(t)}{x_o}) \geq .2$. The break curve in Figure 3 is stochastic in nature and shows an increasing probability that the unit

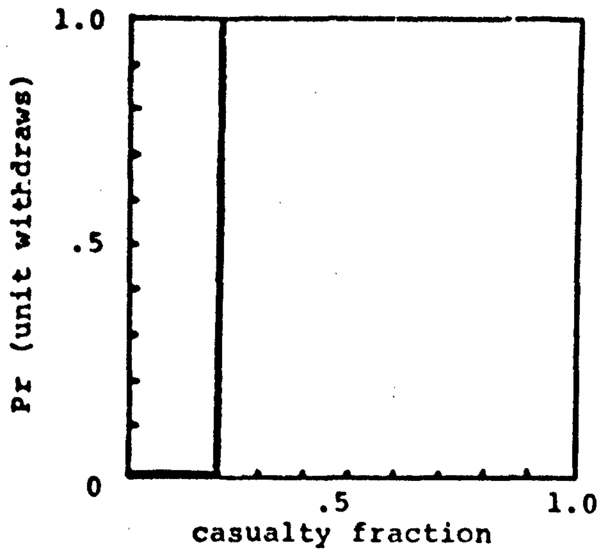


Figure 1. A Deterministic Break Curve.

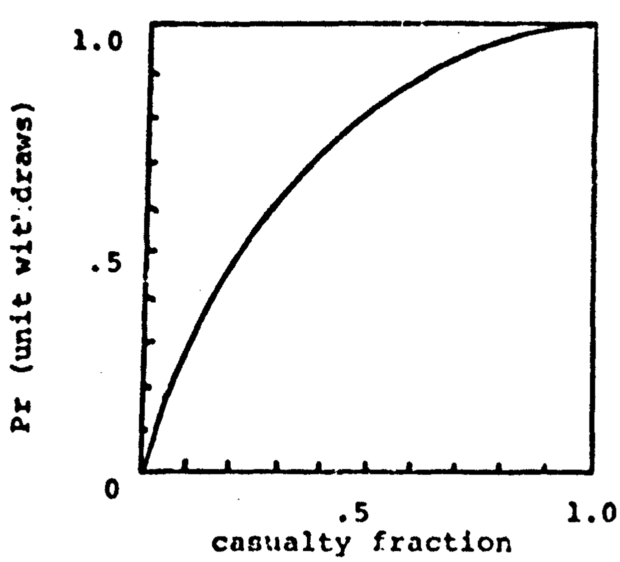


Figure 2. A Stochastic Break Curve.

will stop fighting as its casualty fraction becomes larger. Both types of break curves provide guidance for the assignment of breakpoints to combating units. Of course, the shape of a unit's break curve is based on an estimation of the units tendency to stop fighting as its fractional casualty values change. An important aspect of the break curve model is that a unit's breakpoint is determined by only one variable (fractional casualties). The rationale for such a model is that a unit becomes less effective as its casualty fraction increases and that there is a specified effectiveness level at which the unit will elect to stop fighting. The usefulness of the break curve model is derived from the fact that the number of casualties throughout a battle is relatively easy to obtain from most combat models. The break curve model then appears to be both logical and mathematically tractable. In order to be a good predictive model, however, the model should be valid. In other words, the model should reflect conditions as they exist in actual combat. Validating any combat model is almost an impossible task at the present state of art. The reasons are quite obvious and much too numerous to list here. The results of two attempts to validate break curve models are discussed below.

C. VALIDITY OF BREAK CURVE MODELS

In testing for the validity of a combat model one is almost entirely reliant on data from past conflicts to the extent that the data exists. Even if the collected data

tends to support the hypothesis purported by the model then the statistical questions of reliability, sampling techniques, homogenity, etc. arise and tend to cast shadows of doubt on the conclusions. Granted that doubt still remains in any conclusions on combat model validity, several people have tested actual combat data to see if it supported the break curve models. Clark [Ref. 8] gathered combat data from World War II to see if a deterministic break curve was applicable to participating combat units. Two of Clark's conclusions were:

> 1. The statement that a unit can be considered no longer combat effective when it has suffered a specific casualty percentage is a gross oversimplification not supported by combat data.
>
> 2. The very wide individual differences in the ability of infantry battalions to carry out a given mission cannot be accounted for in terms of casualties alone, no matter how the data are presented.

An excellent study was done by Helmbold [Ref. 13] to test the validity of a stochastic break curve model. A brief description of the assumptions, procedures, and conclusions are worthwhile. Helmbold postulated the following hypotheses:

1. The breakpoint for each side is a random variable from some probability distribution and is independent of the opposing side's breakpoint. Prior to a battle each side randomly and independently chooses a casualty fraction value at which it will withdraw (or break) from the battle. The battle continues until one side reaches its breakpoint.

2. The break curves for each side are generally applicable for all battles in which there is an attacker and defender.

3. The casualties on side X (the attacker) and side Y (the defender) are related in a monotonic increasing manner by some function ϕ. If $f_x(t)$ and $f_y(t)$ are the fraction of casualties on side X and Y respectively at some time t after the start of the battle then $f_x(t) = \phi\,[f_y(t)]$.

Hypothesis (3) necessarily limits Helmbold's conclusions to the case of a deterministic relationship between X and Y casualties. One such case might be a casualty process described by Lanchester's equations that model aimed fire ("square law"). It is also important to note that ϕ is a monotonic increasing function and will have a unique inverse ϕ^{-1}. Helmbold then developed [Ref. 13] the following two relationships between conditional probabilities:

1. $\Delta xx^{(q)} = \Delta yx\ [\psi^{-1}(q)]$
2. $\Delta yy^{(q)} = \Delta xy\ [\psi\ (q)]$,

where

$\Delta xx^{(q)} = P(f_x < q \mid W_x)$,

f_i = factional casualties for side i, $\quad i = X,Y,$

W_i = Side i wins, $\quad i = X,Y,$

$0 < q < 1,$

$yx^{(q)} = P(f_y < q \mid W_x)$, $\psi(q) = \min\ [\phi_{(q)}, 1]$ where

$f_x = \phi(f_y)$.

$yy^{(q)} = P(f_y < q \mid W_y)$,

$xy^{(q)} = P(f_x < q \mid W_y)$

Knowing formulae (1) and (2) Helmbold was able to determine $\psi(q)$ and $\psi^{-1}(q)$ by plotting actual historical combat data. An illustration of the technique used is shown below.

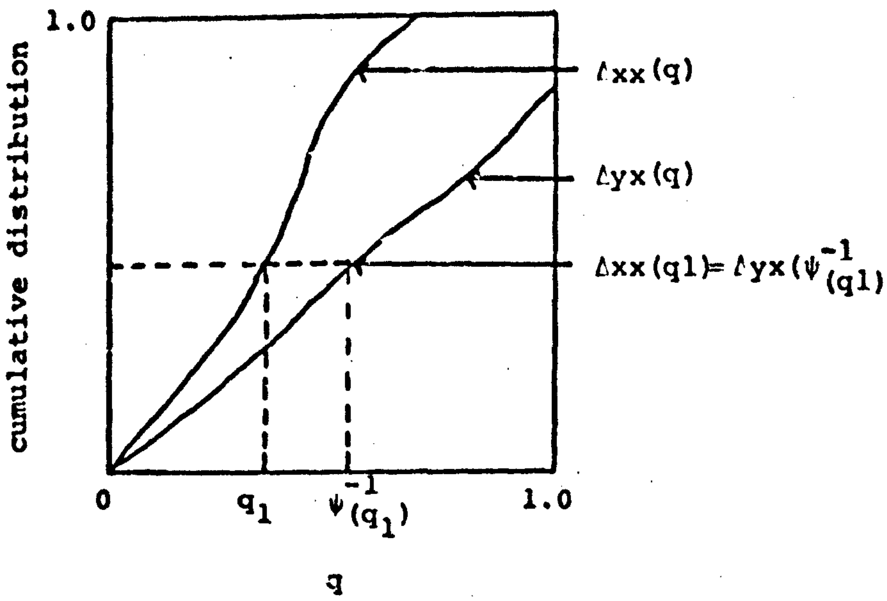


Figure 3. Casualty Fraction Distribution in Battles Won by Attacker (X).

When data on all battles is collected it is first broken down into battles won by attacker (X) and battles won by defender (Y). Under each of these two categories the battles are listed according to percentage of casualties for X and percentage of casualties for Y. By tabulating the data in this manner it is possible to plot the cumulative distribution functions shown in Figures 3 and 4. Notice in the figures that the winner's CDF should plot above and to the left of the loser's CDF. For any probability value it is possible to read values of q and $\psi^{-1}{}_{(q)}$ from Figure 1 and q and $\psi_{(q)}$ from Figure 2. By repeating this process for

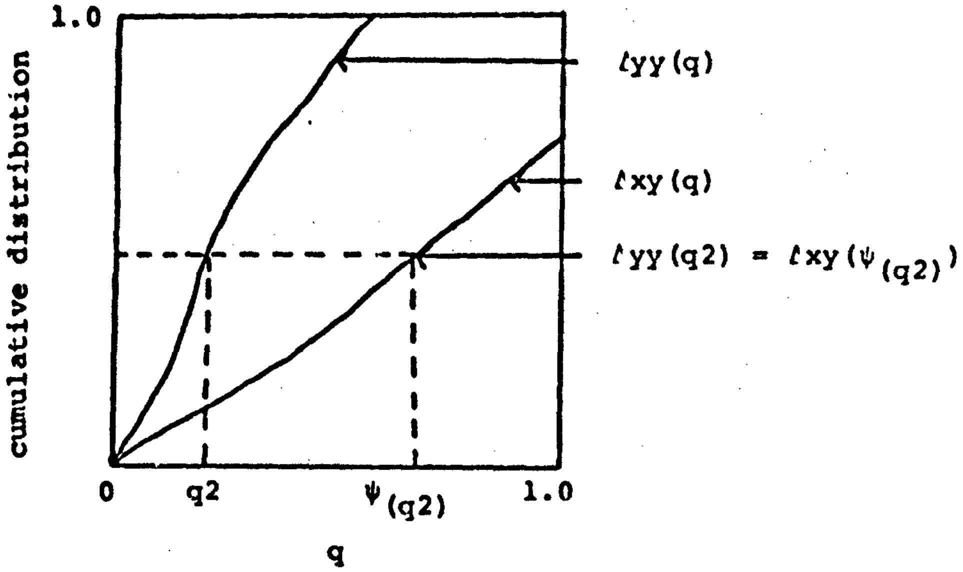


Figure 4. Casualty Fraction Distribution in Battles Won by Defender (Y).

numerous probability values it is possible to estimate the functions ψ and ψ^{-1}. When ψ and ψ^{-1} are plotted for argument values between 0 and 1 the resulting plot should closely resemble that of a function and its inverse. An example of a function and its inverse plotted between 0 and 1 is given in Figure 5. The inverse function should be a mirror image of the function in the 45 degree line through the origin.

When Helmbold plotted the actual historical data he found that ψ and ψ^{-1} did not demonstrate a true inverse functional relationship. With this motivation Helmbold made the following conclusion: "Consequently, it seems that the soundness of models of combat that make essential use of breakpoint hypotheses must be considered suspect until a

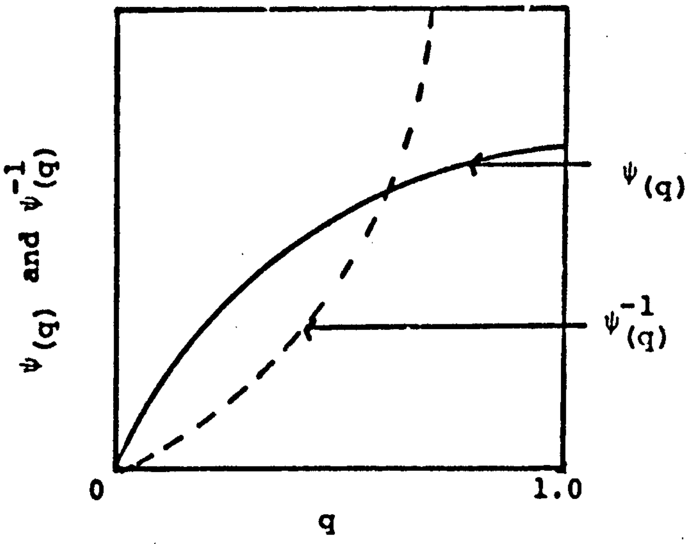


Figure 5. An Example of Inverse Functional Relationship.

better theoretical understanding of the battle termination process is obtained."

In reviewing Helmbold's study one might postulate several reasons why historical data does not support the three breakpoint hypotheses. The first and most obvious reason is that one or more of the hypotheses are not true. For example, it is possible that the breakpoints of X and Y forces are not independent but depend on some variable such as force ratio. The possibility also exists that there is no functional relationship between X and Y casualties (i.e., casualties occur randomly). Even if the hypotheses are accepted as true there are still numerous possible reasons for historical data disagreement. Several possible reasons are listed below.

1. It is possible that the casualty data from each battle is, at best, a gross estimate; although, this is partially compensated for by the large (1080) sample of battles if one can assume that the mean error of estimation is near zero.

2. It is possible that the two sides in the battle should not have been classified as attacker and defender. For example, other classifications might have been larger force vs. smaller force or most combat experienced force vs. least combat experienced force. The latter classification would possibly be quite difficult.

3. It is possible that an exogenous variable such as time dictates that the sample of battles be drawn from the same time period. The rationale might be that military doctrine and hardware change so drastically over time that it is not proper to include all battles in one population.

The possibilities listed above are by no means exhaustive. They are listed only to illustrate possible additional considerations in Helmbold's study and will not be investigated further.

D. ALTERNATIVES AVAILABLE

If it is true that present breakpoint models that are based on casualty fractions are not valid then there are essentially two alternatives available. One might attempt to modify existing models, or one might attempt to develop new models. Helmbold [Ref. 13] suggested three possible

modifications to the breakpoint models that are currently being used. He concluded that none of the modifications were satisfactory to the extent that they would be desirable breakpoint models. Naturally, the possibilities for further modifications are still open and should not be completely disregarded.

At this point in time, however, it seemed appropriate to develop a new model based on a different approach. In looking for a new approach it was important to become as familiar as possible with the actual process that was being modeled. Primarily there were three sources of information available from which one could gain familiarity with the breakpoint process. They were personal experience, subjective opinions from military personnel, and the literature. After evaluation of information available it was concluded that the breakpoint process is some type of decision-making process and should be modeled as such.

III. APPROACHES TO MODELLING THE BREAKPOINT AS A DECISION PROCESS

In all battles the commanders of participating units are forced to make decisions in the face of uncertainty. There is virtually no possibility that a commander could predict with certainty the state of nature that might exist at some future time in the course of the battle. The reasons for this are obvious: there are too many variables required to describe the state of nature completely, and the values assumed by many of the variables are stochastic rather than deterministic.

A decision process under uncertainty might be described briefly in the following manner. There are a number of courses of action from which the decision maker (i.e. commander) must choose one. Associated with each course of action are possible consequences. The decision maker must determine from pre-selected criteria the desirability of obtaining each consequence. The course of action chosen by the decision maker will be based on both the desirability and probability of obtaining the associated consequences. The risk associated with choosing a course of action may be thought of as the probability of obtaining undesirable consequences.

For the military decision maker there are many problems associated with the process described above. One may not always be able to execute a course of action according to

plan, but in many situations the chosen course of action is revised during execution because of unexpected events. All the consequences associated with a course of action are not known and therefore not considered. Even if the consequences are considered, it is very difficult to assign the probabilities of obtaining them. Many times the military decision maker must consider a course of action not only in terms of consequences but also in terms of future courses of action. Of course, the commander in battle faces many other problems such as time constraints, communication failures, confusion, etc.

Despite all the problems associated with military decision making under uncertainty, research has been conducted in this area, and some limited conclusions have been reached. In a study done by Krumm, Robins, and Ryan [Ref. 19] subjects were tested on their ability to make tactical military decisions. It was hypothesized that the quality of tactical military decision making is a function of the decision maker's experience, his ability, his decision process pattern, and the facts made available to him. The results of the study confirmed this hypothesis. When scores were assigned to the four variables listed above and subjects were tested on their decision making ability the predictor variable which alone accounted for nearly half the common variance among test scores was the subject's decision process pattern. The implications of the results led the authors to state [Ref. 19] "If indeed the manner in

which a subject approaches a problem situation (his decision process pattern) is related to decision quality, then such a relationship should hold for a variety of problem situations. And if such generalization is supported, then it should be possible to improve decision quality in general by educating individuals in systematic problem solving techniques." These conclusions suggest that prior training in problem solving techniques influences the quality of decisions made by military commanders.

The U.S. military services provide both doctrine and guidance for making sound military decisions, and it is assumed that military services of other nations provide the same. Of course, to assume that one could model an individual's decision process pattern based strictly on doctrine and guidance provided him by his military service is not very realistic. However, there are certain aspects of military doctrine which should provide some insight into the commander's decision process pattern. For example the doctrine stating that "the accomplishment of the mission is most important" should influence a commander's decisions to the extent that he will decide on those courses of action which he feels will result in accomplishment of the mission. On the other hand, one sould not expect the commander to decide on courses of action which he felt would minimize casualties but result in failure of the mission. This was a simple example but it shou serve to illustrate how knowledge of doctrine and past training might provide some

insight into the military decision process. Indeed it would be negligent to ignore the effects of doctrine and past training when considering a model that involved military decision making.

A. ANALYSIS OF MILITARY DECISION MAKING

At this point it will be beneficial to discuss in general terms some of the doctrine and guidance which might affect the decisions made by a U.S. Infantry commander. When assigned a mission it is necessary for the commander to operationally define the mission in terms of concrete and well-defined objectives. Then all efforts and assets are directed toward the objectives. Some typical objectives might be to capture and secure a piece of terrain or to fix the enemy in place by denying him freedom of movement. The The procedure used by the commander to designate objectives and decide on courses of action which will attain those objectives is referred to as "the estimate of the situation" [Ref. 30]. The procedure follows five basic steps.

1. Mission

The first step involves studying the mission to determine what tasks must be performed to accomplish it.

2. Situation and Courses of Action

The second step involves gathering in an orderly manner all facts which are relevant to the situation. If facts are not available then logical assumptions are made. All information gathered is used to determine factors which may affect any possible course of action, to determine

opposing conditions which may adversely affect the accomplishment of the mission, and finally to formulate possible courses of action.

3. Analysis of Opposing Courses of Action

The third step involves determining probable events that will occur during the execution of courses of action when faced with opposing conditions.

4. Comparison of Own Courses of Action

The fourth step involves evaluating the advantages and disadvantages of each course of action and choosing those courses of action which promise to be most successful in accomplishing the mission.

5. Decision

The fifth step involves choosing a course of action and translating it into a complete statement as to the action to be taken.

In both analyzing such a situation and postulating approaches to modeling the situation it is necessary not only to limit the scope but also to make certain assumptions. This allows one to formulate a situation which is somewhat more mathematically tractable.

Consider a situation in which a friendly and enemy infantry company are operating independently in some sector of terrain. Suppose that both companies have been given the general mission of locating and destroying any opposing forces in the particular sector of terrain. Further assume that there is a meeting engagement between the two companies.

This implies that the element of surprise is equally distributed between the two companies. At this point the following questions seem appropriate: Will both units elect to commit themselves to a decisive engagement at the time of initial contact? If both units commit themselves how long will the battle last? Finally, at what point, if ever, in the battle will one side elect to withdraw from the battlefield and leave the other side in control?

Before attempting to provide answers for the above questions it is beneficial to analyze, in a formal manner, the decision processes of both commanders. The analysis required the identification of the decision variables, the relevant state variables, the relationship between state and decision variables, and the decision criteria. Possibly the first decision to be made by the commander is whether or not to decisively engage the enemy unit upon initial contact. The decision the commander makes may be thought of as the outcome of a single Bernoulli trial: the variable can assume only one of two possible values corresponding to the decision to decisively engage and the decision not to decisively engage. The variable will assume the two values with probability p and 1-p respectively. The value of p is not known and must be estimated.

The probability that the commander will decide upon decisive engagement is postulated to be a conditional probability which is conditioned on values assumed by relevant state variables. In other words, the probability that

the commander will decide to decisively engage the enemy will not always be the same in every situation but will change according to his perception of relevant state variables. For example one would not expect the probability of a company commander deciding to engage an enemy squad to be the same probability of deciding to engage an enemy battalion when the commander was aware of the size of the enemy units. In the example relative size of the enemy force would be a relevant state variable since it is used to describe the state of an existing system. The state of the system might be thought of as "the minimum amount of present information about the history of the system which allows one to predict the effect of the past upon the future" [Ref. 34].

The first problem is to attempt to identify the variables which describe the state of the system. For example in some tactical situations the actions of adjacent friendly units must be considered as a state variable but in other situations, such as the meeting engagement described above, there are no adjacent friendly units to be considered. If one can successfully list all state variables to be considered then the problem becomes one of determining the degree of importance of those variables. For example a state variable such as trafficability of terrain might be relevant for an attacking tank unit but might not be so relevant for an air mobile assault. At first glance one might conclude that identification and classification of all relevant state variables is an impossible task. In full context the conclusion might

be justified, however a general knowledge of the tactical situation and reliance on military doctrine and expertese may permit some of the relevant state variables to be identified.

In the meeting engagement example some of the relevant state variables which could influence the probability of either commander deciding on decisive engagement are the mission of the unit, the perception of relative force size, the relative tactical posture of opposing forces, the amount of ammunition remaining, and the ability to communicate with subordinate units. In the example the state variable which should have the most influence on the commander's decision to decisively engage the enemy is the mission to locate and destroy all enemy forces in the area. Of course, if one company had almost no ammunition remaining and its platoons were separated by long distances then the commander might decide to wait until a later time to become decisively engaged.

The state variables describe the state of nature at any point in time, and the value of any particular state variable might or might not be relevant to the commander's decision process. According to U.S. Army doctrine the state variables which should be relevant are those state variables influencing the unit's capability to perform the mission. The same statement might not generally be applicable to all other armies in all tactical situations because of differences in doctrine, training and motivation. In such cases the

relevant state variables would have to be determined accordingly. An example might be an army whose doctrine stated that a force ratio of at least three to one must be establsihed before decisive engagement. In this case force ratio would be the important state variable that determined the commander's decision criterion.

In order to identify the relevant state variables in combat one must have as a minimum general knowledge of the tactical situation and the doctrine of units involved. This leads into the next problem area which is determining the relationship between the relevant state variables, the decision criteria, and the decision variables. As mentioned previously any existing relationship between state and decision variables must be established within the context of a generally known tactical situation and doctrine. Assuming that accomplishment of the mission was the overriding decision criterion in the meeting engagement of the two companies which were operating independently and further assuming that both commanders elected to decisively engage the enemy then the relevant state variables become those that affect the capability to accomplish the mission. This implies that the extent to which a state variable is relevant is the degree to which it will influence the unit's capability to accomplish its mission.

Whether or not a unit was capable of mission accomplishment can be determined only after the fact, but during a mission it is possible to subjectively estimate whether or

not a unit is capable of accomplishing its mission. Such an estimate could be based on observations of the values of the relevant state variables which affect unit capability and predicting success or failure. A prediction of failure would hypothesize that at some time in the future the unit would become completely noncapable of mission accomplishment. Assuming that a commander is primarily concerned with mission accomplishment and that he constantly assesses his unit's capability to accomplish the mission then the relationship between state and decision variables becomes clearer. The observed values of the relevant state variables provide an estimation of the unit's capability, and the estimation of the unit's capability influences the commander's decision. For example a relevant state variable such as ammunition remaining is observed to be zero. The commander estimates that his unit is not capable of mission accomplishment and therefore decides to withdraw his unit from the battlefield. Usually the case is never as simple as the example, but the relationship is the important concept to be stressed.

In the example of the two companies which experienced a meeting engagement it should be appropriate now to discuss when the battle might end and with what results. Clearly the battle would end if one of the companies were annihilated or if both sides simultaneously withdrew from the battlefield. The case of interest however is the one in which one company withdraws and leaves the other company in control

of the battlefield. In light of previous discussion it is postulated that a commander would decide to withdraw (i.e., reach the breakpoint) when he estimated that his unit was no longer capable of accomplishing its mission. In other words the decision criterion is the commander's estimate of his unit's capability. As long as the commander estimates that his unit is capable of accomplishing the mission he will continue to engage the enemy.

The exact process by which a commander estimates the unit's capability is naturally very complex and differs from individual to individual, but there are common steps in the process which might be analyzed. For instance the estimated values of relevant state variables contribute in estimating capability. The commander must also predict future values of the relevant state variables based on past and present observations in order to visualize the final outcome of the battle. For example, if the commander observes that 60% of the people in his unit are casualties and his unit has been taking casualties at the rate of 1% of the initial force per minute then he would have good reason to estimate that his unit would be annihilated in forty minutes provided all other state variables remained constant. The important point to be emphasized is that the commander's estimate of the unit's capability was not only based on present values of relevant state variables but also on past and predicted values. This might suggest that the existence of trends in changing state variables is considered by the commander when making estimates.

Of course the commander does not make his estimate of the unit's capability based entirely on observations of changing state variables. He is not always capable of observing all relevant state variables, and he is aware that his perception of values of state variables might differ drastically from the true values. This leads to reliance on such factors as past experience and intuition for making estimates on unit capability. Such factors are difficult to quantify, and it will be assumed in later development that estimates of unit capability are made strictly from estimated values of relevant state variables. There are many state variables which could possibly influence a unit's capability to perform its mission. The values of some of the variables are relatively easy to quantify while others are almost impossible to quantify. A list of some of the factors that might be considered by a commander when estimating the unit's capability is as follows:

1. Mission and associated objectives

2. Number of casualties and number of key personnel who are casualties

3. Rate at which casualties are occurring

4. Availability of critical supplies

5. Availability of communications with subordinate units and higher HQs

6. Force ratio of friendly and enemy combatants

7. Relative tactical posture of friendly and enemy combatants

8. Availability of intelligence on enemy intentions
9. Training and experience level of friendly combatants
10. Fatigue and motivation
11. Proportion of reserves committed
12. Status of adjacent units
13. Weather and terrain conditions
14. Availability of reinforcements and supporting fires
15. Availability of means to evacuate and treat casualties

The list above is by no means exhaustive, and the variables listed are not all independent since the change in the value of one variable could necessarily mean a change in value of another variable. The list does illustrate the fact that there are numerous state variables which could be relevant in estimating a unit's capability for mission accomplishment.

In the meeting engagement example the company commander woudl monitor numerous state variables and continually evaluate the relative effectiveness of the two opposing companies. Based on predicted changes in values of the state variables the commander would estimate the probability that the unit was capable of accomplishing the mission. If the commander estimated that there was a relatively low probability of mission accomplishment then he might consider three alternative courses of action. The first alternative would be to continue the mission at all costs. The second would be to continue the mission for a period of time, make another estimate, and consider the possible courses of action again. The third would be to break contact with the

enemy and withdraw from the battlefield. This process is analagous to that of sequential testing in which a tester either accepts or rejects a given hypothesis or elects to continue testing until more information is available. If the battle progressed until the commander estimated that there was virtually zero probability of mission accomplishment then at some point in time he would be faced with deciding on essentially two courses of action. He could decide to continue the mission until annihilation or withdrawal from the battlefield. If one assumes that accomplishment of the mission was important to both companies in the meeting engagement example and at some point in time during the battle one of the companies decided to withdraw from the battlefield, then the company that withdrew is said to have reached its breakpoint.

There are several possible approaches that could be taken to model a unit's breakpoint as a rational decision process. One could assume that the combat dynamics were unknown and that the decision to "break" was a result of estimating the values of state variables and projecting those estimates into the future. On the other hand, one could assume that a model for the combat dynamics was generally applicable but that certain parameters in the model were unknown. The decision to "break" could be based on estimates of the unknown parameters and projections of those estimates into the future. In any approach that is taken one should not neglect to consider the fundamentals

of military decision making and the types of combat situations which adapt themselves to the possible existence of breakpoints. In the next sections two general approaches to modelling the breakpoint as a battle termination decision will be discussed.

B. FIRST APPROACH: EXTRAPOLATION OF OBSERVATIONS WITH NO ASSUMPTION ABOUT COMBAT DYNAMICS

In describing an approach to modelling the breakpoint as a result of extrapolation of empirical observations the following assumptions are applicable.

1. Decisions are based on the perception of a future state.

2. The decision maker has perfect knowledge of relevant state variables.

3. The relevant state variables completely describe the state space.

4. A functional relationship exists between the state variables and a unit's capability to perform a specific task.

5. The combat dynamics are unknown.

The implication of this type of approach is that the state variables or a function of these is the decision criterion which causes a commander to stop fighting. The justification for this is that changes in state variables cause changes in a unit's capability to perform specified tasks.

At this point it is useful to introduce the concept of a capability index denoted by CI. The values of CI are restricted such that $0 \leq CI \leq 1.0$. When CI = 1.0 the state space

is assumed to be ideally favorable for accomplishing a task. This implies that there is no enemy resistance, the friendly unit is functioning perfectly, the weather and terrain are ideal, etc. Naturally the CI value of exactly 1.0 is more of an abstraction than a reality, but it does provide an origin for a scale to measure relative capability. When CI = 1.0 the probability of accomplishing the task in the most desirable manner is unity. This, of course, implies that the probability of mission accomplishment is also unity. If CI = 0 the state space is assumed to be ideally unfavorable for accomplishing a task. This implies that conditions are such that the probability of accomplishing the task is zero. A value of CI = 0 would be appropriate for a unit which had been completely annihilated.

From assumptions three and four it is possible to express CI as:

$$CI = \emptyset \; (X_i(t_n)) \qquad i = 1,\ldots,K$$

where $\emptyset$ undefined function and $X_i(t_n)$ is the relevant state variable i at time t_n. By using the variable t_n it is assumed the time axis is divided into equal segments and a discrete value is observed at the end of the n^{th} interval. From Assumption 1 it is necessary not only to obtain a value of CI at time t_n but also to obtain an estimated value of CI at some time t_{n+1} in the future. An expression for the estimate of CI is denoted as $\hat{CI} = \emptyset \; (X_i(t_{n+1}))$.

One method for obtaining estimated values for the state variables is exponential smoothing. Exponential smoothing

is appropriate for the following reasons:

1. Estimates are based on past and present observations.

2. The method allows most recent observations to be weighed heavily in formulating estimates.

3. The method does not require an exact time history to be carried forward.

The exponential smoothing model to be used is

$$X_i(t_n) = A_i(t_n) + E_i(t),$$

where

$$X_i(t_n) = \text{rate of change of } X_i \text{ per time period}$$

$$= X_i(t_n) - X_i(t_{n-1}),$$

$$A_i(t_n) = \text{constant at } t_n,$$

and

$$E_i(t) = \text{random noise (error) with zero mean.}$$

From Assumption 5 the value of $A_i(t_{n+1})$ is not known and must be estimated. This might be accomplished by using the smoothing function of the observations which is

$$S[\Delta X_i(t_{n+1})] = A_i(t_{n+1}) = \alpha[\Delta X_i(t_n)] + (1-\alpha)\ S[\Delta X_i(t_{n-1})]$$

where α is a smoothing constant and $0 \leq \alpha \leq 1$. A higher value of α will assure a more rapid response to a real change in the pattern of the observations. A smaller value of α will assure a less rapid response. In a combat situation with changing state variables a higher value of α would most likely be appropriate.

An example of exponential smoothing used to predict the change in force ratio (X/Y) is given below in Figure 6. The estimation started at time period 2, and the α value used was 0.9

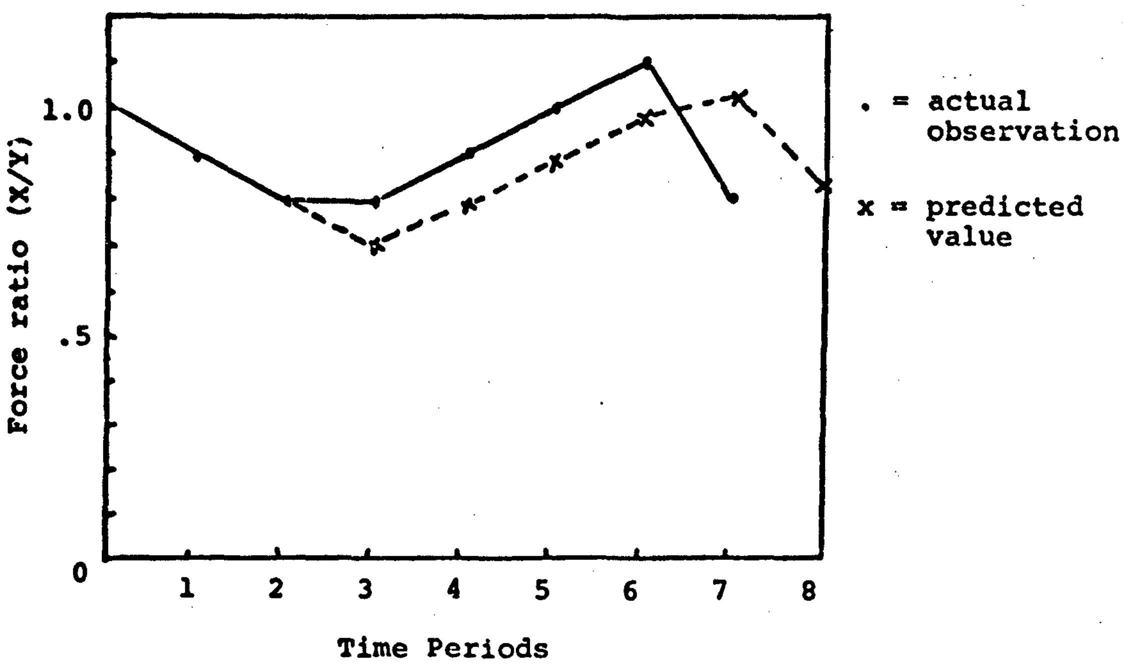


Figure 6. Actual and Estimated Force Ratios.

Assuming such a technique could be used for predicting values of all state variables then CI could be computed if Ø were a known function. The determination of the functional form for Ø is beyond the scope of this thesis. One would suspect that some type of weighting scheme would be necessary to give more weight to those state variables that were most relevant. Also, in some cases the percentage change in state variables during the course of the battle might be considered a better measurement than absolute values.

Investigation of an explicit form for $\emptyset$ might require evaluation of subjective data gathered from a large number of military experts and quantitative data from field exercises. Even then the evaluation would have to be done in light of specific situations and doctrine. If one were to consider only a few quantifiable state variables then estimates for $\emptyset$ would possibly be easier to obtain. Assuming that in a simple case $\emptyset$ could be specified then it would be possible to compute $\hat{CI}$. After finding a value for $\hat{CI}$, where $0 \leq \hat{CI} \leq 1$, then the probability of mission accomplishment can be expressed as a conditional probabil' y, conditioned on the value of $\hat{CI}$. In some situations it is reasonable to assume that the variable known as mission accomplishment (MC) can assume only one of two possible mutually exclusive values. Let a value of 1 indicate that the mission is accomplished and a value of 0 indicate that the mission was not accomplished. Further it is reasonable to assume that the $Pr(MC = 1/CI = 1)$ approaches unity and $Pr(MC = 1 \mid CI = 0)$ approaches 0. The $Pr(MC = 1 \mid CI = z)$, where $0 \leq z \leq 1$, could be plotted in such a manner as that illustrated in Figure 7.

Of course the exact curve is unknown, but the general shape of the curve might resemble the one in Figure 7. If a commander's primary objective in combat is mission accomplishment then it is reasonable to assume that tactical decisions are made in light of such an objective. This should provide the criteria for deciding when a unit reaches

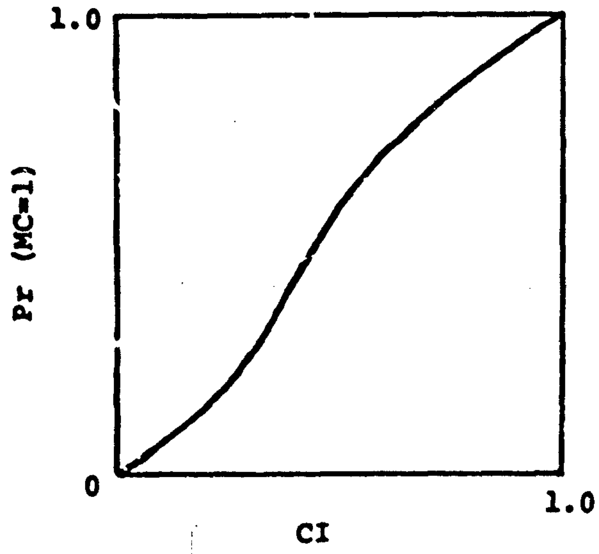


Figure 7. The Probability of Mission Accomplishment.

its breakpoint. If the commander is convinced that the probability of mission accomplishment was very small then he would be more likely to withdraw his unit from the battlefield than if he was convinced that the probability of mission accomplishment was very high.

Figure 8 graphically illustrates the probability of a unit reaching its breakpoint as the probability of mission accomplishment changes. Once again the exact shape of the actual curve is unknown, but the general shape of the curve in Figure 8 is intuitively appealing for several reasons:

1. It illustrates low and high probabilities for reaching a breakpoint when the probability of mission accomplishment is respectively high and low.

2. It also illustrates a rapid change in the probability of unit breaking when the probability of mission accomplishment changes from values above .5 to values below .5. A

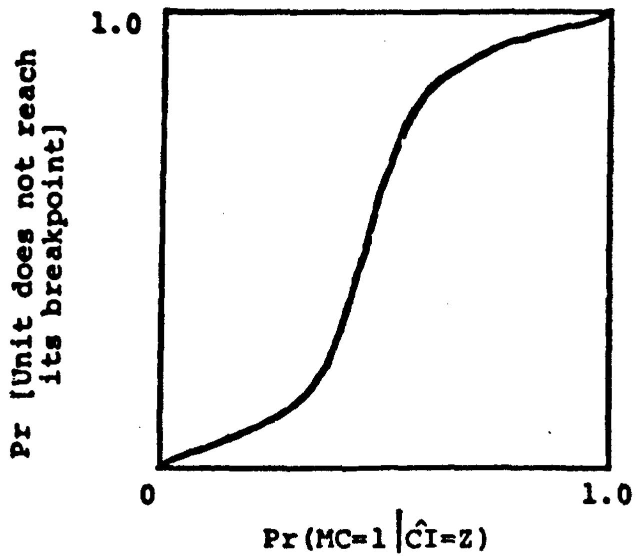


Figure 8. The Probability that a Unit Does Not Reach Its Breakpoint.

curve such as the one in Figure 8 would provide a means for determining whether or not a unit had reached its breakpoint. The following steps outline a procedure to model the breakpoint as a rational decision process based on extrapolation of observed state variables.

1. For a given unit choose several of the most relevant state variables based on the tactical situation and the applicable doctrine.

2. Determine how much the unit's capability to perform an assigned task is influenced by a change in the state variables, and derive a function $\emptyset$ such that $CI = \emptyset\ [X_i(t_n)]$.

3. At fixed time intervals during the battle compute $\hat{CI}$ by using exponential smoothing to predict values for $(X_i(t_n)$.

4. For a value of CI determine a value of $P_r(MC = 1)$ from a graph such as the one in Figure 7.

5. For a value of $P_r(MC = 1 / \hat{CI} = z)$ determine a value

of P_r (unit does not reach its breakpoint) from a graph such as the one in Figure 8.

6. For the value of P_r (unit does not reach its breakpoint) compare a probability value obtained from a random number generator.

7. Based on the comparison decide whether or not the unit reached its breakpoint.

A numerical example illustrating these procedures is presented in Appendix A. The advantages of using an approach such as the one described above to model a unit's breakpoint are listed below.

1. The approach allows more than one state variable to influence a commander's decision.

2. The approach requires no prior knowledge of the actual combat dynamics.

3. The approach allows for decisions based on prior, present, and predicted observations of state variables.

4. The approach is flexible in that it can be modified to fit numerous tactical situations.

5. The approach considers the element of chance by introducing a stochastic decision rule.

6. The approach is computationally simple and could easily be handled by a computer.

7. The approach is oriented toward current military doctrine.

The disadvantages of such an approach are listed below.

1. It might be very difficult to obtain an explicit function that describes a unit's capability in terms of the relevant state variables.

2. The approach provides a positive probability that both sides in a battle might reach their breakpoint at the same time. Although this is not completely unrealistic it is a rare event.

3. The assumption that the commander has perfect knowledge of all relevant state variables is not realistic.

4. The approach does not account for unquantifiable state variables such as fear, morale, experience, etc.

As is the case with most approaches to modelling combat the most difficult aspect is that of investigating the validity of proposed models. To determine the validity of the models hypothesized above, it would be necessary to do more research in the area of military decision making in combat. The specific area of research that is crucial to justifying the proposed approach is the sensitivity of the decision variable to changes in state variables. Such an investigation would necessarily require that subjective data be gathered from military commanders. One method of gathering data might be to present general tactical situations to commanders and simulate a battle by specifying the values of state variables at regular time intervals. At the end of each time interval the commander would be required to decide on either continuing the battle or

withdrawing his unit. Each time a decision was made the commander would be required to explain his decision in terms of the decision criteria. This method would provide a starting point for identifying those state variables which the commanders considered relevant and the degree of relevancy associated with each one. Of course, any conclusions drawn from the data would be applicable only to the same type of situations that were presented to the commanders.

Although the method described above would not serve to validate a model it would create a sound military basis for assigning weights to relevant state variables. An obvious caveat in utilizing simulated combat conditions to analyze military decision making is that decisions are made with the realization that no real losses will occur. Since a commander is not conditioned to readily admit that his unit is no longer capable of accomplishing an assigned mission the data gathered in any simulation would possibly reflect a bias because of such conditioning. Any approach that required the use of historical data to investigate the validity of the proposed approach to modelling the breakpoint would probably not be feasible because of data insufficiency. With the introduction of computer-based tactical data systems on the battlefield it could be possible in the future to acquire more data on relevant state variables and attempts at validation of the breakpoint model based on historical data might be justified.

At this point it might be appropriate to mention a possible modification that could be inforporated into the previous approach described above. Rather than assuming that the combat dynamics are unknown, one could assume that the state variables change with time according to some known system dynamics although there is some error associated with each process. The error could arise from two possible sources: (1) the assumed model does not adequately describe the true process and therefore such errors might be considered as systematic or bias errors, and (2) actual observations of state variables and are considered purely random errors. In order to make an estimate of the true process and subsequently make predictions of the future states of the process with postulated dynamics it is necessary to use a procedure generally referred to as a filtering technique [Ref. 18]. At this point it is sufficient to mention the availability of a technique which can be used to estimate and predict the true state of a process which is considered deterministic in nature. In the next section a special type of filtering commonly known as Kalman filtering will be discussed in the context of an assumed attrition model.

C. SECOND APPROACH: EXTRAPOLATION BASED ON ASSUMED COMBAT DYNAMICS

The second approach to modelling the breakpoint as a decision process required the following assumptions:

1. The battle dynamics are described by a form of Lanchester's equations.

2. The values of the attrition-rate coefficients have been determined by considering significant tactical variables.

If there were no uncertainties in the equations describing the battle dynamics, the commander's decision on whether or not to continue the battle would be relatively easy to determine for any specified criteria. For example, if Lanchester's equations for "modern warfare" were applicable and the X commander stated that he would continue to fight if and only if the opposing unit reached its breakpoint before his unit was annihilated then the decision rule could be developed in the following manner.

Let $\frac{dx}{dt} = -ay$ and $\frac{dy}{dt} = -bx$ describe the battle dynamics: where $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are the rates over time at which the X and Y forces change,

a and b are constant attrition-rate coefficients,

and

x(t) and y(t) are X and Y forces at time t.

Solving the two differential equations simultaneously, the time solutions are:

$$X(t) = (X_o - Y_o\sqrt{a/b})\frac{e^{\sqrt{ab}\,t}}{2} + (X_0 + Y_o\sqrt{a/b})\frac{e^{-\sqrt{ab}\,t}}{2}$$

and

$$Y(t) = (Y_o - X_o\sqrt{b/a})\frac{e^{\sqrt{ab}\,t}}{2} + (Y_o + X_o\sqrt{b/a})\frac{e^{-\sqrt{ab}\,t}}{2},$$

where X_o and Y_o are the forces at time t = 0 when the commander of the X forces makes a decision.

If Y_{BP} is the force level at which the Y unit will reach its breakpoint then

$$Y_{BP} = (Y_o - X_o \sqrt{b/a}) \frac{e^{\sqrt{ab}\, t_{BP}^{Y}}}{2} + (Y_o + X_o \sqrt{b/a}) \frac{e^{-\sqrt{ab}\, t_{BP}^{Y}}}{2},$$

where t_{BP}^{Y} = time at which Y unit reaches its breakpoint. Solving for t_{BP}^{Y} results in the following expression:

$$t_{BP}^{Y} = \frac{1}{\sqrt{ab}} \ln \left\{ -Y_{BP} + \frac{\sqrt{Y_{BP}^2 + b/aX_o^2 - Y_o^2}}{X_o \sqrt{a/b} - Y_o} \right\}.$$

Using the same method the time at which the X force level goes to 0 denoted by t_o^X can be solved for:

$$t_o^X = \frac{1}{2\sqrt{ab}} \ln \left\{ \frac{X_o + Y_o \sqrt{a/b}}{X_o - Y_o \sqrt{a/b}} \right\}.$$

The decision rule then can be expressed in the following manner:

$$\begin{cases} \text{If } t_{BP}^{Y} < t_o^X & \text{continue the battle.} \\ \text{If } t_{BP}^{Y} \not< t_o^X & \text{do not continue the battle.} \end{cases}$$

The case of no uncertainties in the battle dynamics is straightforward, and the decision rule will depend only on the decision criteria specified by the commander.

The more important and relevant cases are those in which there exist uncertainties. Several cases will be considered in which different variables or parameters in each case are stochastic in nature. The first case to be considered is a meeting engagement between X and Y forces,

and the commander of the X forces must decide whether or not to decisively engage the Y forces. Assume that the mission of the X forces is of such a nature that the option of decisive engagement is left entirely to the X commander. This case may be thought of as a unit deciding to terminate battle or reach its breakpoint at time t=0. In other words, the commander would estimate the probability of winning the battle and base his decision on that estimation. The decision criterion would be the probability that X wins. If the estimated probability were small then the commander would be less likely to decide on decisive engagement with Y. Assume once again that the model is Lanchester's equations for "modern warfare." Further assume that the attrition-rate coefficients are known but that X_f (force level at which X forces become completely ineffective), Y_f (force level at which Y forces become completely ineffective), and Y_o (the initial force level of Y) are random variables with known distributions. The probability that X wins can be developed as follows:

$$\frac{dx}{dt} = -ay \quad \text{and} \quad \frac{dy}{dt} = -bx,$$

where

U_f = force level at which U becomes completely ineffective, U=X,Y

and

t_f^U = time at which U becomes completely ineffective, U=X,Y.

$$P_r(X \text{ wins}) = P_r(t_f^Y < t_f^X) = P_r\left(\left(\frac{1}{\sqrt{ab}}\ \ln\left\{\frac{-\sqrt{a}\ Y_f + \sqrt{aY_f^2 + bX_o^2 - aY_o^2}}{\sqrt{b}\ X_o - \sqrt{a}\ Y_o}\right\}\right) < \left(\frac{1}{\sqrt{ab}}\ \ln\left\{\frac{-\sqrt{b}\ X_f + \sqrt{bX_f^2 + aY_o^2 - bX_o^2}}{\sqrt{a}\ Y_o - \sqrt{b}\ X_o}\right\}\right)\right).$$

The above can be shown to be equivalent to:

$$P_r(X \text{ wins}) = P_r\ \left[(bX_f^2 - aY_f^2) < (bX_o^2 - aY_o^2)\right] .$$

Let $X_f = F^X X_o$ and $Y_f = F^Y Y_o$ where $0 \leq F^i \leq 1 \quad i = X, Y$

Let $Z_o = X_o / Y_o$.

Then $$P_r(X \text{ wins}) = P_r\left(Z_o^2 > \frac{a(1-(F^Y)^2)}{b(1-(F^X)^2)}\right) = P_r\left(\frac{(1-(F^X)^2)}{(1-(F^Y)^2)}\ (Z_o)^2 > \frac{a}{b}\right)$$

$$\triangleq P_r\ (W > a/b)$$

From previous assumptions Z_o, F^Y, and F^X are random variables with known distributions. To find a single distribution which would characterize the probability statement above is difficult; however if one were to assume distributions for Z_o, F^Y, and F^X and use Monte Carlo methods to estimate a distribution for W, the P_r (X wins) could be obtained for different values of a/b. This has been done and the results are shown in Appendix B.

It is worthwhile to discuss the appropriate distributional forms for Z_o, F^Y and F^X. If one assumes that the combatants operate as units then a discrete distribution would be appropriate for Z_o. For example if X_o is a company

then Y_o might be a squad, a platoon, a company or a battalion. This would suggest that $Z_o = X_o/Y_o$ would assume only one of four possible values. The probabilities associated with each of the values would require either intelligence on the general deployment of Y forces or frequencies of unit sizes engaged in the past or both. The distributions of F^X and F^Y realistically should be discrete in nature but could be approximated by continuous distributions when X_o and Y_o are not extremely small. The distributions should be such that the probability of obtaining values of F^X and F^Y close to 0 or 1 is relatively small since once units are decisively engaged they rarely reach their breakpoint at $X_{BP} = X_o$ or $X_{BP} = 0$. Since negative values of F^X and F^Y are not feasible, distributions from the gamma family might be appropriate. In any event the P_r (X wins) can be calculated [Appendix B] for different values of a/b. The P (X wins) can be used as a factor which will determine the value of the decision variable. For example, the X commander might specify a criteria such that P (X wins) $\geq .5$ means he will always engage and P (X wins) $< .5$ means that he will never engage. This would be a deterministic decision process since a particular value of a/b (or greater) will always insure that P_r (X wins) $\geq .5$.

Another decision criteria might be P_r (X decisively engages Y) = P_r (X wins). In this case for a known value of a/b the P_r (X wins) is also known, and the procedure to decide whether or not X decisively engages Y could be made

randomly. This procedure is indicative of a stochastic decision process. Of course it is possible that the commander might specify other criteria in formulating his decision to engage. For example the expected loss ratio could be significant in reaching a decision, especially if $E[F^Y] << E[F^X]$ or vice versa. The decision criteria then would specify a minimum acceptable expected loss ratio as well as an acceptable probability of winning. Appendix B illustrates how the expected loss ratio could be computed and incorporated into the decision rule. The situation described above and illustrated in Appendix B considered a model which had three unknown and two known parameters. The technique used to derive appropriate decision rules is also applicable to other analytic models with all or some of the parameters unknown. For example, the probability of winning and the expected loss ratio could have been computed for the above model even if Z_o, F^X, F^Y, a and b were all unknown. The only restricting aspect of such an approach is the capability of assigning appropriate distributions to the unknown variables. The availability of data from past battles and intelligence on current activities and capabilities of combat units would definitely influence the capacity to develop appropriate distributions.

It must be remembered that the previous discussion has dealt with a method for determining the probability that a commander would decide to engage an enemy when certain parameters in a specified model are unknown. This could be

considered a special case of a general combat situation where a model is assumed but the parameters within the model are unknown. A more general situation is a case when the commander not only has to decide whether or not to initiate battle but also has to decide how long he will continue the battle. If a general model for the combat dynamics is assumed, then the commander's problem would be one of estimating parameters and predicting outcomes. The estimation of parameters in the model would be based on past and present observations of state variables by the commander.

At this point it is hypothesized that the longer the commander observes (i.e. the more observations made at equally spaced time intervals) the state variables, the estimates for model parameters become closer to the true parameter values. If one can assume that the commander observes the true state of nature then the hypothesis above is statistically appealing since, generally, the variance of estimated parameters that are stable in time decreases as sample size increases. Realistically, in battle a commander may not have to depend on repeated observations to get an accurate estimate for certain unknown parameters. The reason for this is a combination of experience and obvious cues presented. For example the size of an enemy force can often be determined in a short period of time by noting both the volume of fire and the organic weapon support peculiar to specific sized units. Variables such as experience and battle cues, however, are difficult to quantify and include

in any type of model. For this reason the approaches for estimation of unknown parameters and prediction of future outcomes discussed later will not explicitly account for such variables. If one could assume that the combat attrition process could be described by Lanchester's equation for "modern warfare" but that the parameters in the model were unknown then the commander of each opposing side would try to estimate the value of the unknown parameters in order that he might predict future states in the battle. Of course ultimately the commander wants to predict the final outcome of the battle in order that he might choose an appropriate course of action. There are several techniques that could be used to model the estimation of unknown parameters and the prediction of future states of the battle.

If one could assume that the general form of the combat dynamics is known for a particular type of battle but that the measurements or observations of state variables are subject to random error (Gaussian), then an appropriate technique for estimating the true state of nature and predicting a future state of nature might be Kalman filtering [Ref. 27].[1] This technique gives unbiased estimators and can be considered as a modern version of Gauss' least-squares technique. Consider a dynamical system or process whose state can be characterized by vector difference equation:

$$\underset{\sim}{X}(K+1) = \underset{\sim}{f}(\underset{\sim}{X}(K)K,K),$$

$\underset{\sim}{X}(K)$ is an n-dimensional state vector at time K,

---

[1]This approach was suggested by H. K. Weiss (see also [Ref. 39]).

and $\quad \underset{\sim}{f}$ is an n-dimensional vector function.

Also consider a vector measurement equation:

$$\underset{\sim}{X}(K) = \underset{\sim}{g}(\underset{\sim}{X}(K), \underset{\sim}{V}(K), K),$$

$\underset{\sim}{Z}(K)$ is an q-dimensional output vector at time K,

$\underset{\sim}{V}(K)$ is an q-dimensional vector of random measurement noise at time K,

and $\quad \underset{\sim}{g}$ is an q-dimensional vector function.

The model above might be used to describe state variables changing in a discrete manner with time and observations being made at discrete time intervals. The Kalman filtering technique is applied to the model to give an estimate of the state of nature at time K and a prediction of the state of nature at time K+1. An example of how the Kalman filter might be applied to estimate and predict the values of parameters in Lanchester's equations for "modern warfare" is given in Appendix C.

The application of the Kalman Filer allows one to estimate and predict the state of nature for a model with unknown parameters and continually update the estimates and predictions by considering the most recent observations. This suggests that the Kalman filtering technique might be very useful in obtaining predictions of relevant state variables which in turn might influence decisions that are based on a perception of the future state of nature. If one assumes that the general manner in which the relevant state variables change is known and that decisions are influenced by a

perception of the future then the breakpoint might be modelled as a random decision variable whose probability of realization varies according to predicted future states. The decision criteria could be expressed in terms of an estimated capability index ($\hat{CI}$) that was explained and used in Section III-B. This would require the same type of probability curve that was illustrated in Figure 8.

D. PROBLEMS IN MODELLING A DECISION PROCESS

At the beginning of this section several problems related to military decision making were discussed. The problems associated with modelling the military decision process are just as numerous and complex. A primary source of all the problems is the inability to completely describe the decision process in an operational manner such that every aspect is clearly understood. There are certainly elements in the process which are common to most so-called "rational" decision makers, but individual traits and preferences are likely to be influential to such an extent that a generalized description of the process is not feasible.

Although research is being conducted on tactical military decision making [Ref. 19], the full extent to which such factors as prior experience and training influence the decision process is not realized. One area of current interest in information systems is individual preference for different amounts and types of data when presented with a decision task. The element of stress also influences individuals in varying degrees to the extent some people

become irrational when making a decision under stress. To completely account for the factors mentioned above in a decision model is not feasible at the present state of the art. The attempts to gain further insight into military decision making in combat are hampered by such things as the availability of data, the inability of commanders to recall concrete reasons for making certain decisions (i.e. decisions based on feelings, hunches, etc.), and the inability to account for the influence of personal interactions on the battlefield. The validity of any model that postulates the same deterministic decision process for more than one individual would certainly be suspect in light of the problems mentioned. Possibly a better approach would be to try to account for individual differences in the decision process by allowing each decision to be a random variable as was suggested previously in the approaches to modelling a breakpoint as a decision process. Hopefully, the probabilities associated with values of the decision variable can be estimated better through further research and testing of military decision making.

IV. POSSIBLE APPLICATIONS OF BREAKPOINT DECISION MODELS

To model the breakpoint as a decision process is intuitively appealing. Historically the time at which a unit has disengaged from the enemy and withdrawn from the battlefield has been decided by the commander of that unit. Naturally there are exceptions, but in general disciplined units follow instructions issued by the commander. If it is the commander who decides when the unit has reached its breakpoint then it is worthwhile to consider how he makes this decision when establishing battle termination rules for combat models.

A. COMPUTER SIMULATIONS

The first type of models in which the approach described in Section III-B might be applied are computer simulations. Extrapolation of observations with no assumption about combat dynamics could be used as a sub-model in both low resolution, highly aggregated simulations such as ATLAS [Ref. 17] and high resolution simulations such as DYNTACS [Ref. 9]. The calling sequence for the sub-model during the simulation could be handled in several different ways. One way would be to require the sub-model to be called at fixed time intervals; another would be strictly event oriented (i.e. called when relevant state variables changed values): and another would be a combination of time and event oriented calling sequences.

Since a computer simulation is well suited for storing large quantities of information it is capable of monitoring the changing values of a large number of state variables. This means that all the simulation input variables that were quantifiable and considered relevant to the unit's capability to perform could be monitored and the unit's estimated capability index (CI) could be computed easily by including a routine for exponential smoothing in the software. The formulae for computing CI would necessarily be adjusted to account for the tactical situation and the doctrine of opposing forces. Exact formulation of CI and the probability of reaching a breakpoint for given values of CI might be based on results of subjective data from military experts and field experimentation. In any case further research is required here. The value of the decision variable could then be determined stochastically by using any one of several random schemes.

It is particularly important to consider breakpoints when one wishes to make an assessment of the final outcome of a battle which has involved many units in sustained combat. It is quite possible that an outcome of such a battle is very sensitive to the termination rules imposed on participating units. This could be verified by varying the termination rules for several runs of the simulation and comparing final outcomes. One advantage of using a decision type approach to determine breakpoints is that one is forced to constantly monitor the state of nature described by the

state variables. This could prevent unrealistic events such as a unit continuing to fight for two hours without any ammunition. In general computer simulations would be highly adaptive to including a sub-model of battle termination based on a stochastic decision process.

B. ANALYTIC MODELS

In a deterministic model the outcomes at various phases of battle can be determined exactly before the battle begins. Establishing breakpoint termination rules involves only the specification of appropriate decision criteria. In other words, no new information is gained through observing the battle process over time, and the prediction of future states in the battle will be realized exactly. The specification of breakpoint decision criteria can be based on the desirability of a realized final outcome of the battle. This of course is not realistic and implies that an all or nothing tactic would be optimal if a specific set of results were acceptable and all others unacceptable. The decision process under complete certainty is not very applicable to a realistic decision making environment such as land combat.

If one considers a more realistic situation in which there is uncertainty in a model, then the modelling of a unit's breakpoint becomes more involved. The commander's decision criteria often rely on estimates of unknown variables or parameters, and predictions of outcomes which may or may not be realized. In combat situations where a general type of analytical model is assumed to be applicable but the

values of some or all of the parameters are not known and must be estimated from observations, a technique such as Kalman filtering might be very useful for modelling a commander's estimations and predictions. The estimations and predictions obtained through Kalman filtering then could be used to form the basis for a commander's decision, and one could essentially combine estimation with optimization of battle outcome. This might be accomplished by using the predicted values of relevant state variables to estimate a predicted capability index ($\hat{CI}$) and further, determine the probability of the unit reaching its breakpoint. The rationale is the same as the approach described in Section III-B, but the technique for obtaining predictions is different, and one is not forced to assume that the decision maker's observations correspond exactly with the true state of nature.

When using a filtering technique such as the Kalman filter, it is desirable to use a computer as a computational aid. With the use of a computer and Kalman filtering one could develop an adaptive sub-model (with stochastic elements) for battle termination.

C. PLANNING MILITARY OPERATIONS

In planning for any military operation one would like to know how much resistance the enemy will offer in order to allocate resources accordingly. This requires one to estimate when the enemy will reach a breakpoint (i.e. the duration of the battle) and what factors will cause it. If good

predictive models of breakpoints can be developed the resource allocation problem should become less complex. In order to develop good breakpoint models the process by which a unit reaches its breakpoint should be understood more clearly and then modelled accordingly. The approach of modelling the breakpoint as a stochastic decision process based on the commander's estimate of the unit's capability to perform its mission is recommended as a possible approach. If adequate models can be developed from this approach then the applications in planning for resource allocation are many indeed.

V. DISCUSSION

The primary reason for suggesting that new approaches for modelling the breakpoint are needed is the importance of battle termination sub-models in: (1) determining outcomes in large unit combat models, and (2) optimizing time-sequential tactical decisions. The intent of this analysis has been to postulate alternate approaches to modelling a breakpoint and to suggest reasons why previous approaches were not adequate. It is certainly not disputed that casualties are a very important consideration in determining breakpoints, but it is felt that any good predictive model must consider other state variables as well.

Another important point in considering a model for breakpoints is the extent to which a unit's actions are influenced by the commander's decisions. If it is true that the commander decides when the unit should stop fighting and withdraw from the battlefield, then it is reasonable to investigate possible reasons and the criteria on which the commander bases his decision. It has been postulated in this analysis that one of the primary reasons a commander might decide to stop fighting and withdraw from the battlefield is a conclusion, reached by estimation and prediction, that the unit is no longer capable of accomplishing its mission. This is stated without evidence but is based on prevailing doctrine which stresses the importance of mission accomplishment.

VI. SUMMARY

In this thesis the concept of a unit's breakpoint has been investigated with emphasis on breakpoints of relatively small infantry units. The problems with models that determine a unit's breakpoint strictly from fractional casualties were discussed, and reasons were given for suggesting new possible approaches to modelling the breakpoint. The rationale for modelling the breakpoint as a decision process was discussed and two approaches for such modelling of the breakpoint were proposed. The first approach was postulated in the context of unknown battle dynamics. The second approach assumed that the general combat process was known but that there existed some uncertainties in the model. Specific examples of both approaches were presented. Possible applications for models developed from the approaches were discussed, and finally, areas for further study were recommended.

APPENDIX A

AN EXAMPLE OF APPROACH 1

The following example is a simplified illustration of how one could model the breakpoint of a unit by utilizing the approach described in Section III- . Consider a scenario in which two opposing infantry companies have a meeting engagement. Both companies are operating independently, and there are no other units in the area. The mission of both units is to search for and destroy any opposing units in the area. Assume neither side has access to any outside support for the duration of the battle. The terrain and weather are equally favorable to both units. Assume for simplicity that the commander of Company X has identified three relevant state variables which will affect his capability to destroy the opposing Company Y. Let the three state variables be denoted as:

X_1 = force ratio (X/Y)

X_2 = percentage of X force that is capable of maneuvering

and

X_3 = relative tactical posture of X forces compared with tactical posture of Y forces.

Let the X commander's capability to perform the assigned mission assume the following functional form:

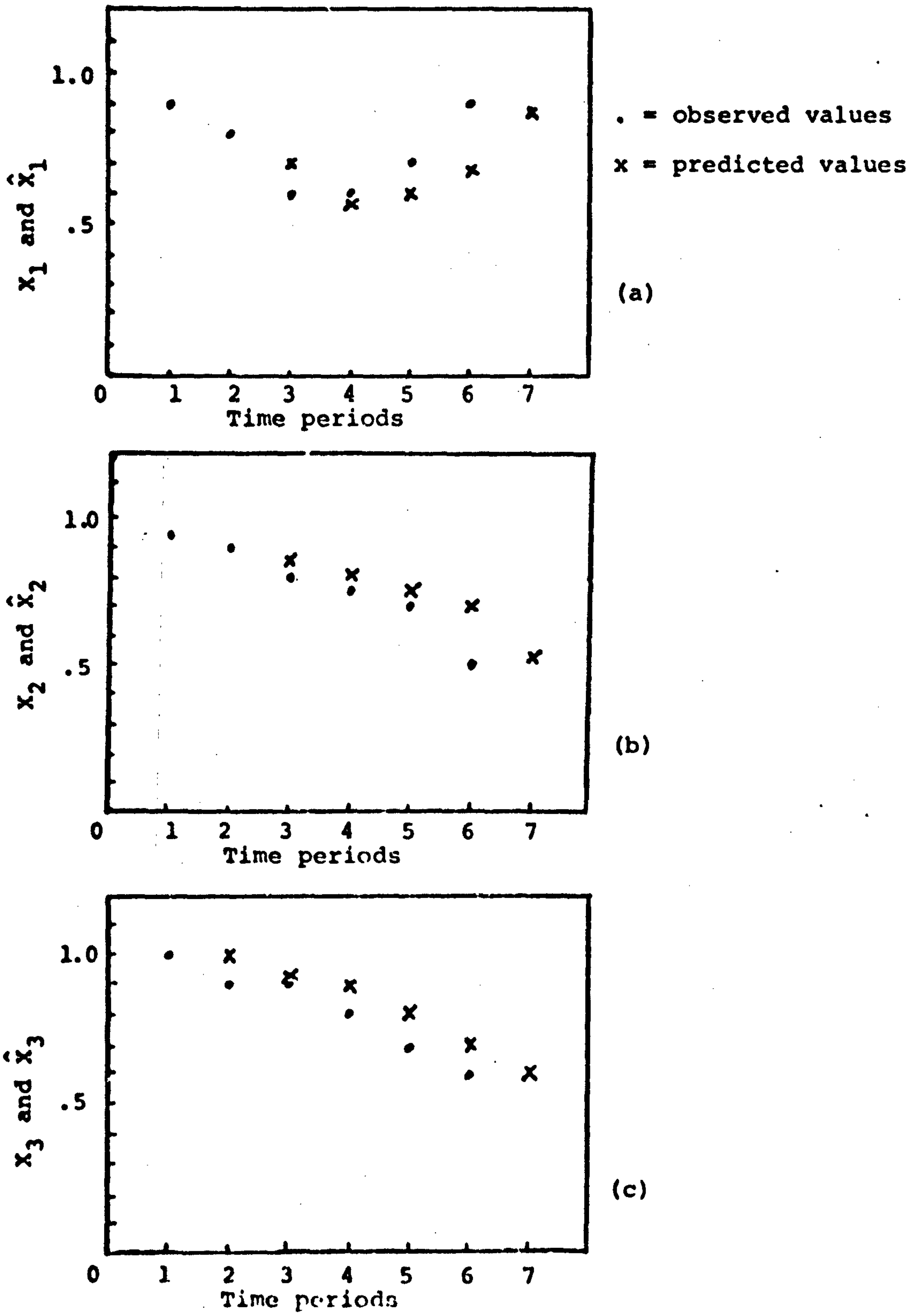
$$CI_X = \begin{cases} 1 - \exp - \left\{X_1 + .2X_2 + .1X_3\right. & X_1, X_2, X_3 > 0 \\ 0 & X_1, X_2, X_3 = 0 \\ & 0 \leq CI < 1 \end{cases}$$

It follows that

$$\hat{CI}_x = \begin{cases} 1 - \exp - \left\{\hat{X}_1 + .2\hat{X}_2 + .1\hat{X}_3\right. \\ 0 \end{cases}$$

$$\hat{X}_1,\hat{X}_2,\hat{X}_3 > 0 \qquad \hat{X}_1,\hat{X}_2,\hat{X}_3 = 0 \qquad 0 \leq CI < 1$$

where $\hat{u}$ = a predicted value of u. From the above formulation it can be seen that the X commander considers X_1 to be the most relevant variable in determining his capability to accomplish the task defined by his mission. Assume that the X commander observes the true value of X_1, X_2, X_3 at equal intervals of five minutes. At the time of each observation he calculates the average change of each state variable over the time interval between observations and predicts future values of the state variables. Based on the predicted values of X_1, X_2, and X_3 he calculates $\hat{CI}$. Suppose that initially $X_1 = X_2 = X_3 = 1$. This implies that the X commander initially estimates CI = .727. Assume that during the first thirty minutes of battle the actual and predicted changes in X_1, X_2, and X_3 are those shown in Figure 9. At the end of each time interval the X commander computes $\hat{CI}$ for the next future time interval, and for each value of $\hat{CI}$ there is an associated probability that the commander will decide that the unit will no longer be capable of mission accomplishment and that the best course of action is to withdraw from the battlefield. Assume that the probability of Unit X reaching a breakpoint for all values less than or equal to $\hat{CI}$ is the cumulative distribution function shown in Figure 10.



NOTE: Predicted values were obtained by using exponential smoothing with a smoothing constant α = .90.

Figure 9. Observed and Predicted Changes in Relevant State Variables.

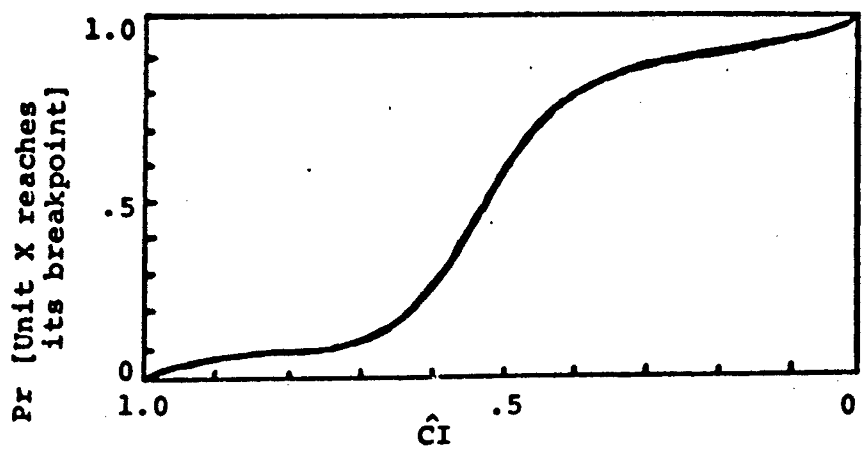


Figure 10. The Probability that Unit X leaches a Breakpoint.

For the first thirty minutes of the battle between X and Y forces the probabilities that Unit X reached its breakpoint at the end of any time interval are shown below in Figure 11.

Time Interval	$\hat{CI}$	Probability that Unit X Reached Its Breakpoint
1	.695	.13
2	.660	.16
3	.617	.20
4	.559	.40
5	.563	.35
6	.593	.30
7	.648	.17

Figure 11. The Probability that Unit X Reached Its Breakpoint.

Assuming that the decision at the end of any time interval is independent of all previous decisions then one can use a random scheme to determine whether or not the X commander decided to withdraw at each time interval. The same type of procedure can be used for determining whether or not unit Y reaches its breakpoint. Of course unit Y might have different relevant state variables, a different formula for computing $\hat{CI}$, different probabilities of reaching a breakpoint when $CI \leq X$, and different time intervals at which decisions are made.

This example was not presented to model a realistic situation but rather to illustrate a procedure that could be used to determine breakpoints when the general form of battle dynamics is unknown and all information is gained by observing values for the state variables.

APPENDIX B

THE DECISION TO DECISIVELY ENGAGE AN ENEMY FORCE

From Section II-D the probability that X wins was expressed as the following:

$$\Pr\,[X \text{ wins}] = 1-\Pr\left[\frac{1-(F^X)^2}{1-(F^Y)^2}\,(Z_o)^2 < a/b\right] \triangleq 1-\Pr(W<a/b)$$

For a specific case assume the destributions of the random variables F^X, F^Y and Z_o can be approximately described as the following:

$$\Pr(F^X \leq X|10) \approx \begin{cases} 0 & X \leq 0 \\ \frac{1}{2}\int_0^X t^2 e^{-t}dt & X > 0, \end{cases}$$

$$\Pr(F^Y \leq X|10) \approx \begin{cases} 0 & X \leq 0 \\ \frac{1}{4}\int_0^X t\, e^{-\frac{1}{2}t}dt & X > 0, \end{cases}$$

and

$$\Pr(Z_o = X) \begin{cases} .3 & X=\frac{1}{3} \\ .4 & X=1 \\ .3 & X=4 \\ 0 & \text{otherwise} \end{cases}$$

The distributions of F^X and F^Y were approximated by gamma distributions with different parameters for each distribution. The distribution of W was not known but could be found approximately by using a monte carlo technique. The

cumulative distribution functions (CDFs) for the three random variables were plotted and three random numbers were drawn from a uniform (o,1) distribution. Then by knowing the probability value associated with each of the random variables the corresponding argument value could be read from the plot of the CDF. Of course, a value for W could then be computed. A special provision was necessarily needed to assure that the values of F^Y and F^X were never greater than or equal to 1.0. This required the distributions of F^Y and F^X be truncated at X = 9.9999... . Ten thousand values were found for W and plotted on a histogram. The frequency plot and CDF for W are shown in Figures 12 and 13 respectively for a range of arguments values from 0 to 3.0.

Given the distribution in Figure 12 it is very easy to compute the Pr (X wins) for different values of a/b. For example when a/b = 1.0 the Pr (X wins) = 1-.69 = 0.31. Knowing the Pr (X wins) two decision rules for decisive engagement can be postulated. The first rule is deterministic in nature and requires the commander to specify a minimum value for Pr (X wins) under which he will never engage and over which he will always engage. For example the commander might indicate that the Pr (X wins) = .5 is his decision criteria for engagement. Then for a specific value of a/b he will either always engage or never engage. A more realistic decision procedure might specify that Pr (X decisively engages) is proportional to Pr (X wins). For example consider the case when Pr (X decisively engages) =

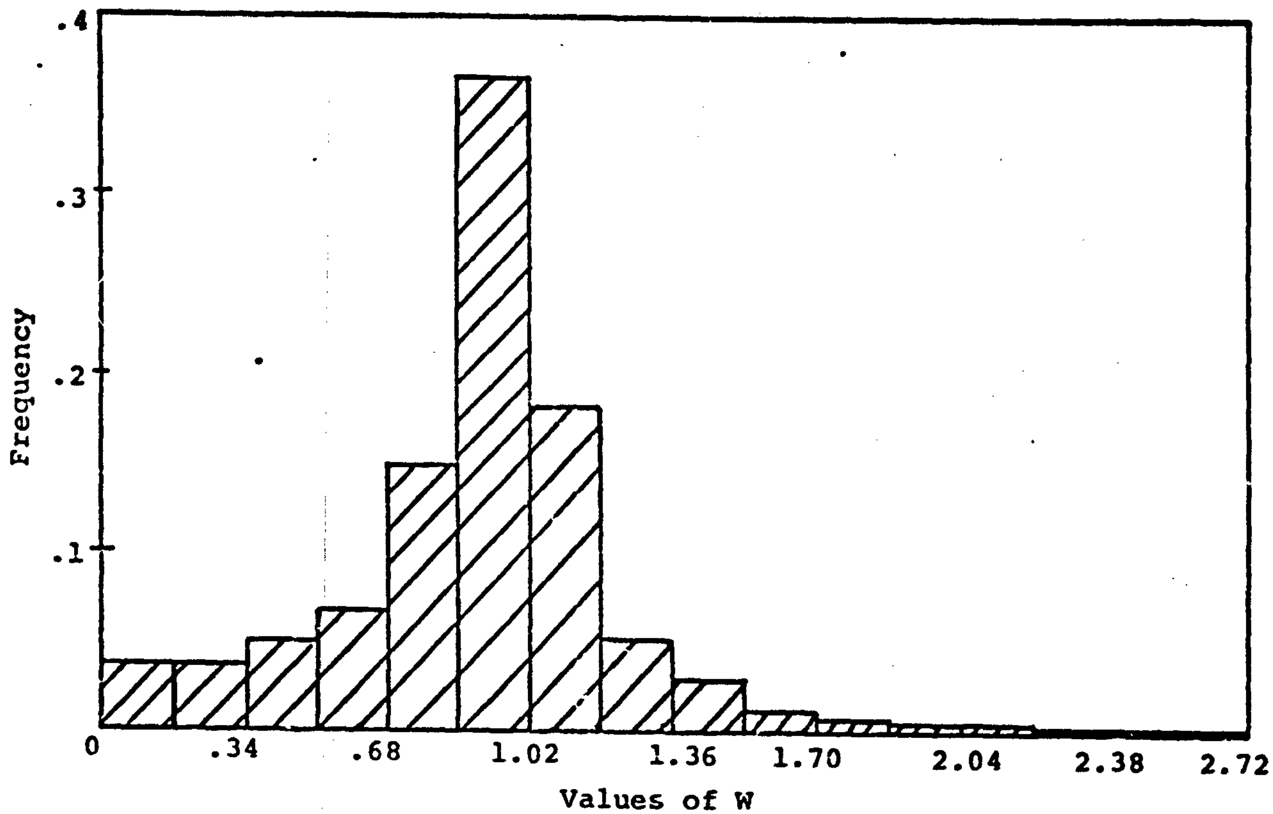


Figure 12. Frequency Distribution of W.

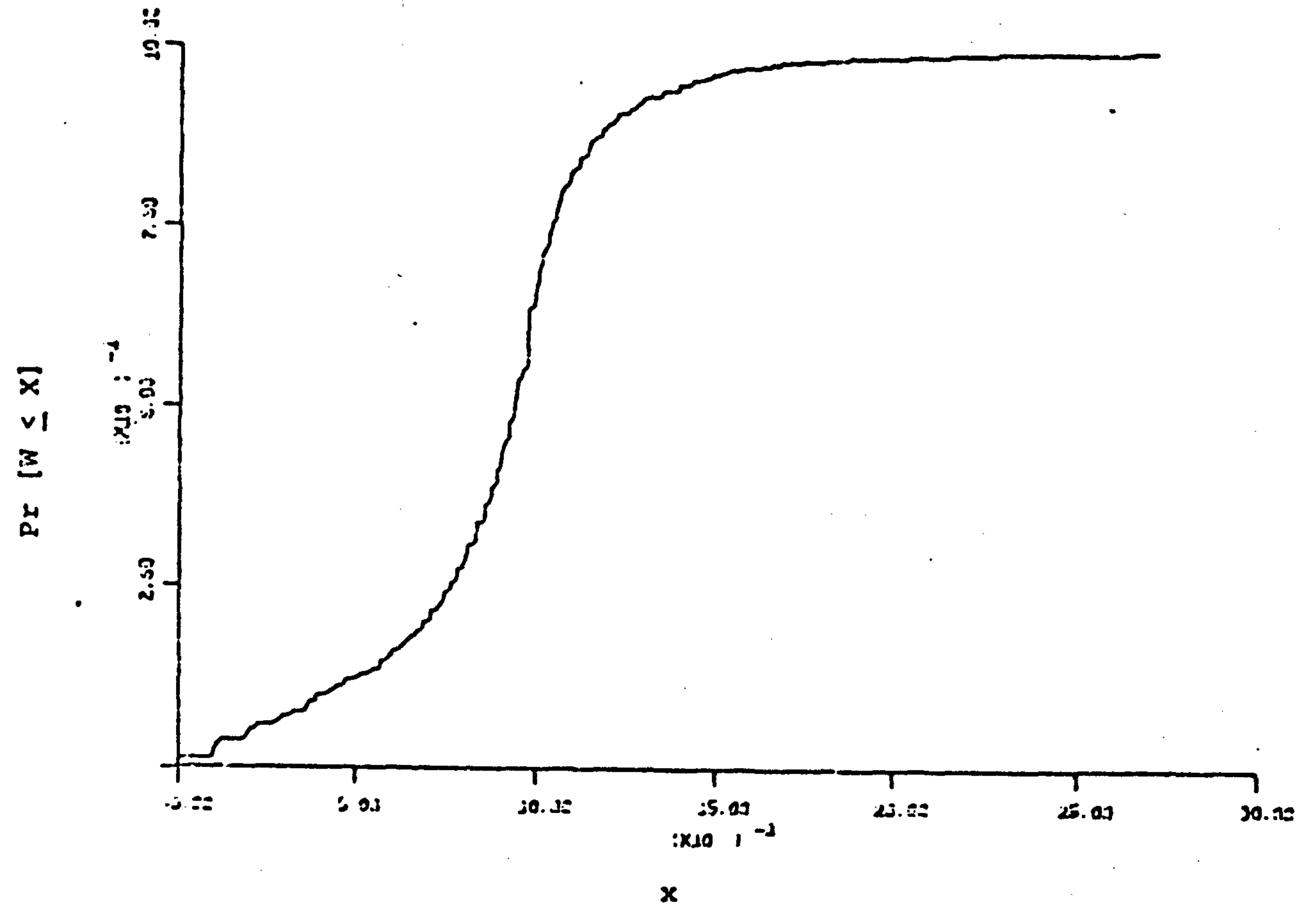


Figure 13. Cumulative Distribution Function for W.

Pr (X wins). In this case a random scheme would be used to determine whether or not the X commander decided to decisively engage the Y forces.

So far the only variable considered in the decision process has been Pr (X wins). Realistically the commander would possibly consider other variables. For example the expected loss ratio $\left(E\left[\frac{F^X}{F^Y}\right]\right)$ might be an important consideration in the commander's decision process. For a case in which the commander's decision was based on Pr (X wins) and $E\left[\frac{F^X}{F^Y}\right]$ a composite criterion would have to be determined. In other words, the commander would have to indicate the relative importance of each variable in his decision criterion. A possible mathematical formulation might be:

$$\text{Pr (X decisively engages)} = \frac{\text{Pr (X wins)}}{E\left[\frac{F^X}{F^Y}\right]}$$

where

$$E\left[\frac{F^X}{F^Y}\right] \geq 1 .$$

This would tend to decrease the Pr (X decisively engages) as the relative expected losses of X increased. The decision rule for determining whether or not the X commander decided to decisively engage the enemy would once again be implemented by some type of random scheme. Other factors might also be included in the same manner as $E\left[\frac{F^X}{F^Y}\right]$ was above. Of course in deriving a formula for Pr (X decisively engages) one must remember that the range of possible values is 0 to 1.0.

APPENDIX C

AN EXAMPLE OF THE KALMAN FILTER

This appendix presents an example of how the Kalman filter might be used to predict future values of parameters in Lanchester's equations for "modern warfare." Consider the following model:

$$\frac{dx}{dt} = -ay \quad \text{and} \quad \frac{dy}{dt} = -bx$$

where a and b are constants.

Denote the variables as follows:

$$x_1 \triangleq x, \quad x_2 \triangleq y, \quad x_3 \triangleq a, \quad x_4 \triangleq b.$$

Then the system of differential equations may be expressed in the following manner:

$$\underset{\sim}{f}(\underset{\sim}{x}(t)) \triangleq \begin{cases} \dot{x}_1 = -x_3 x_2 \\ \dot{x}_2 = -x_4 x_1 \\ \dot{x}_3 = 0 \\ \dot{x}_4 = 0 \end{cases} .$$

Now consider a set of measurement or observation equations such that:

$$\underset{\sim}{z}(\underset{\sim}{x}(t),\ \underset{\sim}{v}(t)) = \begin{cases} z_1(t) = x_1(t)+v_1(t) \\ z_2(t) = x_2(t)+v_2(t) \end{cases}$$

where $z_i(t)$ denotes a measurement of $x_i(t)$ at time t, $i = 1,2,3,4$, and $v_i(t)$ denotes random noise or error in the measurement of $x_i(t)$ at time t.

Assume that $\underset{\sim}{v}(t)$ is a random vector variable with known covariance and further assume that $E[\underset{\sim}{v}(t)] = 0$.

The first thing that should be done in order to apply the Kalman filter is to transform the non-linear continuous-time process denoted by $\underset{\sim}{f}(\underset{\sim}{x}(t))$ to a non-linear discrete-time process. Define $\underset{\sim}{x}(k+1)$ as $\underset{\sim}{a}(\underset{\sim}{x}(k))$ where $\underset{\sim}{a}$ is an n-dimensional state transition vector. Using a Taylor's series expansion:

$$\underset{\sim}{x}(t+\Delta t) = \underset{\sim}{x}(t) + \dot{\underset{\sim}{x}}(t)\,\Delta t + \ddot{\underset{\sim}{x}}(t)\,\frac{(\Delta t^2)}{2} + \text{H.O.T.}$$

where H.O.T. denotes higher order terms.

$$\dot{\underset{\sim}{x}}(t) \triangleq \underset{\sim}{f}(\underset{\sim}{x}(t))$$

$$\ddot{\underset{\sim}{x}}(t) = \frac{d}{dt}\dot{\underset{\sim}{x}}(t) = \frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{x}}\,\frac{d\underset{\sim}{x}}{dt} = \frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{x}}\,\underset{\sim}{f}$$

Therefore $x(t+\Delta t)$ may be expressed as:

$$\underset{\sim}{x}(t+\Delta t) = \underset{\sim}{x}(t)+\Delta t\underset{\sim}{f}(\underset{\sim}{x}(t))+\tfrac{1}{2}(\Delta t)^2\,\frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{x}}\,(\underset{\sim}{x}(t))\underset{\sim}{f}(\underset{\sim}{x}(t))+\text{H.O.T.}$$

Now let $\underset{\sim}{x}(t) \triangleq \underset{\sim}{x}(k)$ and $\underset{\sim}{x}(t+\Delta t) = \underset{\sim}{x}(k+1)$ and drop the H.O.T. This yields the following expression:

$$\underset{\sim}{x}(k+1) = \underset{\sim}{x}(k)+\Delta t\underset{\sim}{f}(\underset{\sim}{x}(k))+\tfrac{1}{2}(\Delta t)^2\,\frac{\partial \underset{\sim}{f}}{\partial \underset{\sim}{x}}\,(\underset{\sim}{x}(k))\underset{\sim}{f}(\underset{\sim}{x}(k))=\underset{\sim}{a}(\underset{\sim}{x}(k)).$$

Now $\underset{\sim}{a}(\underset{\sim}{x}(k))$ can be expressed in matrix form in the following manner:

$$\mathbf{a}(\mathbf{x}(k)) = \begin{vmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{vmatrix} + \Delta t \begin{vmatrix} -x_3(k)x_2(k) \\ -x_4(k)x_1(k) \\ 0 \\ 0 \end{vmatrix} + \frac{1}{2}(\Delta t^2)$$

$$\begin{vmatrix} 0 & -x_3(k) & -x_2(k) & 0 \\ -x_4(k) & 0 & 0 & -x_1(k) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \text{X} \begin{vmatrix} -x_3(k) \;\; x_2(k) \\ -x_4(k) \;\; x_1(k) \\ 0 \\ 0 \end{vmatrix}$$

$$= \begin{vmatrix} x_1(k) - \Delta t \; x_3(k) \;\; x_2(k) + \frac{1}{2}\Delta t^2 x_3(k) \;\; x_4(k) \;\; x_1(k) \\ x_2(k) - \Delta t \; x_4(k) \;\; x_1(k) + \frac{1}{2}\Delta t^2 \;\; x_3(k) \;\; x_4(k) \;\; x_2(k) \\ x_3(k) \\ x_4(k) \end{vmatrix} = \underset{\sim}{x}(k+1).$$

If an initial estimate of $\underset{\sim}{x}(k)$ [denoted by $\underset{\sim}{x}^{o}(k)$] can be obtained shortly after a battle has started then it is possible to express $\underset{\sim}{a}(\underset{\sim}{x}(k))$ as a linear function by expanding $\underset{\sim}{a}(\underset{\sim}{x}(k))$ in a Taylor series about $\underset{\sim}{x}^{o}(k)$ in the following manner:

$$\underset{\sim}{x}(k+1) = \underset{\sim}{a}(\underset{\sim}{x}^{o}(k)) + \left.\frac{\partial \underset{\sim}{a}}{\partial \underset{\sim}{x}}\right|_{\underset{\sim}{x}^{o}(k)} [\underset{\sim}{x}(k) - \underset{\sim}{x}^{o}(k)] + \text{H.O.T.}$$

Let $\underset{\sim}{A}(k) \triangleq \left.\frac{\partial \underset{\sim}{a}}{\partial \underset{\sim}{x}}\right|_{\underset{\sim}{x}^{o}(k)} =$

$$\begin{bmatrix} 1+\frac{1}{2}\Delta t^2 x_3^o(k) x_4^o(k) & -\Delta t x_3^o(k) & -\Delta t x_2^o(k)+\frac{1}{2}\Delta t^2 x_4^o(k) x_1^o(k) & \frac{1}{2}\Delta t^2 x_3^o(k) x_1^o(k) \\ -\Delta t x_4^o(k) & 1+\frac{1}{2}\Delta t^2 x_3^o(k) x_4^o(k) & \frac{1}{2}\Delta t^2 x_4^o(k) x_2^o(k) & -\Delta t x_1^o(k)+\frac{1}{2}\Delta t^2 x_3^o(k) x_2^o(k) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now $\underset{\sim}{x}(k+1)$ can be expressed as the following:

$$\underset{\sim}{x}(k+1) = \underset{\sim}{A}(k)\ \underset{\sim}{x}(k) + \underset{\sim}{a}(\underset{\sim}{x}^o(k)) - \underset{\sim}{A}(k)\ \underset{\sim}{x}^o(k).$$

Since $\underset{\sim}{x}^o(k)$ is assumed known the expression for $\underset{\sim}{x}(k+1)$ is in linear form. Remembering that z(k) was also in linear form, i.e., $\underset{\sim}{z}(k) = \underset{\sim}{c}(\underset{\sim}{x}(k)) + \underset{\sim}{v}(k)$

where $\quad \underset{\sim}{c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

it is possible now to apply the extended Kalman filter to the process defined by Lanchester's equations for "modern warfare."

At this point it is convenient to introduce the following notation:

$\underset{\sim}{x}(k|k)$ is an estimate of $\underset{\sim}{x}(k)$ given that measurements have been made at time k.

$\underset{\sim}{x}(k+1|k)$ is a prediction of $\underset{\sim}{x}$ at time k+1 given that measurements have been made at time k.

$\underset{\sim}{G}(k)$ is a gain matrix at time k.

$\underset{\sim}{P}(k|k-1) = E\left[(\underset{\sim}{x}(k)-\bar{\underset{\sim}{x}}(k))\ (\underset{\sim}{x}(k)-\bar{\underset{\sim}{x}}(k)^T\right]$ is a covariance matrix of estimation error.

$\underset{\sim}{R}(k) = E\left[\underset{\sim}{v}(k)\ \underset{\sim}{v}(k)^T\right]$

For the model developed previously the appropriate Kalman filter equations are:

1. <u>Gain equation</u>

$$\underset{\sim}{G}(k) = \underset{\sim}{P}(k|k-1)\ \underset{\sim}{C}^T\left[\underset{\sim}{C}\ \underset{\sim}{P}(k|k-1)\underset{\sim}{C}^T+\underset{\sim}{R}(k)\right]^{-1}.$$

2. <u>Covariance of estimation error equations</u>

$$\underset{\sim}{P}(k|k-1) = \underset{\sim}{A}(k-1)\ \underset{\sim}{P}\ (k-1|k-1)\ \underset{\sim}{A}^T(k-1).$$

$$\underset{\sim}{P}(k|k) = [I-\underset{\sim}{G}(k)\ \underset{\sim}{C}]\ \underset{\sim}{P}(k|k-1).$$

3. <u>Filter update equation</u>

$$\hat{\underset{\sim}{x}}(k|k) = \hat{\underset{\sim}{x}}(k|k-1) + \underset{\sim}{G}(k)\ \left[\underset{\sim}{z}(k)-\underset{\sim}{C}(\hat{\underset{\sim}{X}}(K|k-1))\right].$$

4. <u>Prediction equation</u>

$$\hat{\underset{\sim}{x}}(k+1|k) = \underset{\sim}{a}(\hat{\underset{\sim}{x}}(k|k)).$$

An example of how one would actually initialize and update the Kalman filter follows. Suppose a battle had been in progress for several minutes and the X command had made estimates of x_1, x_2, x_3, x_4 based on previous measurements at some time k=-1. One would expect the measurement made on $x_1=x(t)$ to be somewhat more accurate than the estimate of $x_2=y(t)$ since the X commander is able to observe his own casualties more closely than those of the enemy. This

insight might provide some rationale for assigning values to the variance associated with the measurements of x_1, and x_2 and initial values of the four state variables x_1, x_2, x_3, and x_4. Now assume that these variances are known and are independent and constant over time. Also assume that $\underset{\sim}{x}^o(0)$ has been estimated such that $E[\underset{\sim}{x}^o(0)] = \hat{\underset{\sim}{x}}(0|-1) = x_1^o, x_2^o, x_3^o, x_4^o$. The covariance matrices are given as follows:

$$\underset{\sim}{R}(k) = E\left[\underset{\sim}{v}(k)\ \underset{\sim}{v}(k)^T\right] = \begin{bmatrix} \sigma_{v1}^2 & 0 \\ 0 & \sigma_{v2}^2 \end{bmatrix} \triangleq \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix}$$

$$\underset{\sim}{P}(0|-1) = E\left[(\underset{\sim}{x}(0)-\bar{\underset{\sim}{x}}(0))(\underset{\sim}{x}(0)-\bar{\underset{\sim}{x}}(0))^T\right] = \begin{vmatrix} \sigma_{x1}^2 & 0 & 0 & 0 \\ 0 & \sigma_{x2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{x3}^2 & 0 \\ 0 & 0 & 0 & \sigma_{x4}^2 \end{vmatrix}$$

$$\triangleq \begin{vmatrix} e & & \\ & f & 0 \\ 0 & g & \\ & & h \end{vmatrix}$$

An application of the Kalman filter equations would give the following results:

$$\underset{\sim}{G}(0) = \begin{vmatrix} \frac{e}{e+a} & 0 \\ 0 & \frac{f}{f+b} \\ 0 & 0 \\ 0 & 0 \end{vmatrix} \qquad \underset{\sim}{P}(0|0) = \begin{vmatrix} (e - \frac{e^2}{e+c}) & & & 0 \\ & (f - \frac{f^2}{f+d}) & & \\ 0 & & g & \\ & & & h \end{vmatrix}$$

$$
\hat{\underset{\sim}{x}}(0|0) = \begin{vmatrix} x_1^o + \frac{e}{e+a}\,(z_1(0) - x_1^o) \\ x_2^o + \frac{f}{f+b}\,(z_2(0) - x_2^o) \\ x_3^o \\ x_4^o \end{vmatrix} \triangleq \begin{vmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{vmatrix}
$$

$$
\hat{\underset{\sim}{x}}(1|0) = \begin{vmatrix} \alpha - \Delta t\gamma\beta + \frac{1}{2}\Delta t^2\gamma\delta\alpha \\ \beta - \Delta t\delta\alpha + \frac{1}{2}\Delta t^2\gamma\delta\beta \\ \gamma \\ \delta \end{vmatrix}
$$

The vector $\hat{\underset{\sim}{x}}(1|0)$ gives a prediction for $\underset{\sim}{x}(k+1|k)$. When measurements were made at time k+1 the filter would be updated by computing $\hat{\underset{\sim}{x}}\ (k+1|k+1)$ in the manner demonstrated above, and another prediction could be made by computing $\hat{\underset{\sim}{x}}\ (k+2|k+1)$.

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